研磨楔形变幅器辐射面对其共振频率的影响*

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本文从波动方程出发,对楔形变幅器的频率方程作了理论推导,将本文所述的变幅器参数代入该方程,用计算机求得这种不锈钢楔形变幅器的辐射面每磨去0.01mm,其共振频率将上升约2Hz,理论值和实验值基本吻合。

关键词:　波动方程,楔形变幅器,频率方程

Effect of grinding the radiation surface of wedged amplitude transformers on resonance frequency

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Based on the wave equation, the frequency formula of the wedged amplitude transformer is studied in this paper. Substituting the known parameters of the amplitude transformer into the formula, the stainless steel amplitude transformer´s resonance frequency will go up 2Hz when its radiation surface is ground off 0.01mm. The theoretical values agree with experimental results probably.

Key words:　wave equation, wedge amplitude transformers, frequency formula

1　引　言

在大功率超声清洗中,变幅器腐蚀是不可避免的。由空化引起的超声腐蚀导致辐射面出现小孔状凹坑,降低了超声辐射强度,影响清洗效果。一个简单的办法就是将变幅器的辐射面磨掉一个薄层(每次约0.2mm),使小孔状的凹坑变浅,超声强度随之提高。由于研磨使得变幅器相对变短,其共振频率也随之提高。本文对超声清洗中常用的楔形变幅器进行讨论,对于设计系统频率跟踪方案及估算变幅器的寿命是完全必要的。

2　理论推导

图1所示的楔形变幅器可分为4段,用同一种材料制成。由于沿 X 方向作开槽加工,可忽略横向振动,将变幅器等效为宽度为 b 的杆状材料处理。用基本波动方程进行理论推导。定义 ξ 为沿 X 方向的位移,$\partial\xi/\partial x$ 为应变,k 为波数。根据波动方程及边界条件可求得 l_0和 l_1的位移及应变表达式为:

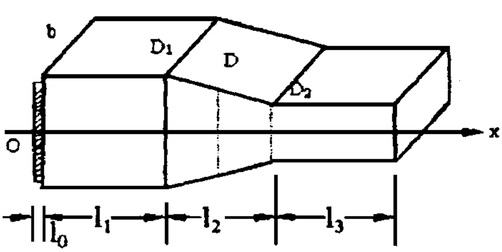

图1　楔形变幅器

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$\xi_0 = \cos kx_0 \quad \frac{\partial \xi_0}{\partial x_0} = -k\sin kx_0$

$\xi_1 = \cos[k(x_1 + l_0)], \frac{\partial \xi_1}{\partial x_1} = -k\sin[k(x_1 + l_0)]$

对 l_2 段:

$D = D_1(1 - \alpha x_2)$, 当 $x_2 = l_2$ 时, $D = D_2$

令 $D_1/D_2 = N$, 则 $\alpha = (n-1)/nl_2$

$S(x_2) = D_1(1-\alpha x)$, $\partial S_2/\partial x_2 = -\alpha b D_1$

代入变截面变幅杆的波动方程可得:

$$\frac{\partial^2 \xi_2}{\partial x_2^2} + \frac{1}{x_2 - 1/\alpha}\frac{\partial \xi_2}{\partial x_2} + k^2\xi_2 = 0$$

令 $z = -k(x_2 - 1/\alpha)$ 代入上式可求得:

$$\frac{\partial^2 \xi_2}{\partial z^2} + \frac{1}{z}\frac{\partial \xi_2}{\partial z} + \xi_2 = 0$$

这是零阶贝塞耳方程, 其通解[1]为:

$$\xi_2 = A_2 J_0[-k(x_2 - \frac{1}{\alpha})] + B_2 N_0[-k(x_2 - \frac{1}{\alpha})]$$

$$\frac{\partial \xi_2}{\partial x_2} = k\{A_2 J_1[-k(x_2 - \frac{1}{\alpha})]$$

$$+ B_2 N_1[-k(x_2 - \frac{1}{\alpha})]\}$$

式中 J_0 是零阶贝塞耳函数, J_1 为一阶贝塞耳函数, N_0 为零阶诺依曼函数, N_1 为一阶诺依曼函数。

由边界条件:

$$\xi_2\Big|_{x_2=0} = \xi_1\Big|_{x_1=l_1} \quad \frac{\partial \xi_2}{\partial x_2}\Big|_{x_2=0} = \frac{\partial \xi_1}{\partial x_1}\Big|_{x_1=l_1} \text{得:}$$

$$\cos[k(l_1 + l_0)] = A_2 J_0[k(\frac{1}{\alpha})]$$

$$+ B_2 N_0[k(\frac{1}{\alpha})] - k\sin[k(l_1 + l_0)]$$

$$= k\left\{A_2 J_1\left[k\left(\frac{1}{\alpha}\right)\right] + B_2 N_1\left[k\left(\frac{1}{\alpha}\right)\right]\right\}$$

由上两式得:

$$A_2 = \frac{\begin{vmatrix} \cos k(l_0 + l_1) & N_0[k(\frac{1}{\alpha})] \\ -\sin k(l_0 + l_1) & N_1[k(\frac{1}{a})] \end{vmatrix}}{\begin{vmatrix} J_0[k(\frac{1}{a})] & N_0[k(\frac{1}{\alpha})] \\ J_1[k(\frac{1}{\alpha})] & N_1[k(\frac{1}{\alpha})] \end{vmatrix}}$$

$$B_2 = \frac{\begin{vmatrix} J_0[k(\frac{1}{\alpha})] & \cos k(l_0 + l_1) \\ J_1[k(\frac{1}{\alpha})] & -\sin k(l_0 + l_1) \end{vmatrix}}{\begin{vmatrix} J_0[k(\frac{1}{\alpha})] & N_0[k(\frac{1}{\alpha})] \\ J_1[k(\frac{1}{\alpha})] & N_1[k(\frac{1}{\alpha})] \end{vmatrix}}$$

对 l_3 段: $\partial^2 \xi_3/\partial x_3^2 + k^2\xi_3 = 0$

$$\text{由边界条件:}\begin{cases} \xi_3\big|_{x_3=0} = \xi_2\big|_{x_2=l_2} \\ \frac{\partial \xi_3}{\partial x_3}\Big|_{x_3=0} = \frac{\partial \xi_2}{\partial x_2}\Big|_{x_2=l_2} \\ \frac{\partial \xi_3}{\partial x_3}\Big|_{x_3=l_3} = 0 \end{cases}$$

得通解为:

$$\begin{cases} \xi_3 = A_3\cos kx_3 + B_3\sin kx_3 \\ \partial \xi_3/\partial x_3 = k[-A_3\sin kx_3 + B_3\cos kx_3] \end{cases}$$

$$\xi_2\big|_{x_2=l_2} = A_2 J_0[-k(l_2 - 1/\alpha)]$$

$$+ B_2 N_0[-k(l_2 - 1/\alpha)]$$

$$\frac{\partial \xi_2}{\partial x_2}\Big|_{x_2=l_2} = k\{A_2 J_1[-k(l_2 - 1/\alpha)]$$

$$+ B_2 N_1[-k(l_2 - 1/\alpha)]\}$$

$$\xi_3\big|_{x_3=0} = \xi_2\big|_{x_2=l_2} = A_3,$$

$$\partial \xi_3/\partial x_3\big|_{x_3=0} = kB_3$$

$$B_3 = \frac{1}{k}\frac{\partial \xi_3}{\partial x_3}\Big|_{x_3=0} = \frac{1}{k}\frac{\partial \xi_2}{\partial x_2}\Big|_{x_2=l_2}$$

$$\begin{cases} \xi_3 = \xi_2\big|_{x_2=l_2}\cos kx_3 + \frac{1}{k}\frac{\partial \xi_2}{\partial x_2}\Big|_{x_2=l_2}\sin kx_3 \\ \frac{\partial \xi_3}{\partial x_3} = -k\xi_2\big|_{x_2=l_2}\sin kx_3 + \frac{\partial \xi_2}{\partial x_2}\Big|_{x_2=l_2}\cos kx_3 \end{cases}$$

由 $\partial \xi_3/\partial x_3 \; {}_{x_3=l_3} = 0$ 即:

$$-k\xi_2\big|_{x_2=l_2}\sin(kl_3) + \frac{\partial \xi_2}{\partial x_2}\Big|_{x_2=l_2}\cos(kl_3) = 0$$

$$\mathrm{tg}kl_3 = \frac{\partial \xi_2/\partial x_2\big|_{x_2=l_2}}{k\xi_2\big|_{x_2=l_2}}$$

$$\mathrm{tg}kl_3 =$$

$$\frac{A_2 J_1[-k(l_2 - \frac{1}{\alpha})] + B_2 N_1[-k(l_2 - \frac{1}{\alpha})]}{A_2 J_0[-k(l_2 - \frac{1}{\alpha})] + B_2 N_0[-k(l_2 - \frac{1}{\alpha})]}$$

上式为整个楔形变幅器的频率方程, 式中 A_2、B_2 已由前面给出。

3 计算结果

根据实验用不锈钢变幅器的尺寸, l_0= 0. 6mm, l_1 = 0.042m, l_2 = 0. 041m, l_3 = 0.042m, $k = 2\pi f/c$, c = 5000m/s, D_1 = 0.042m, D_2= 0.020m, $n = D_1/D_2$ = 2. 1 $\alpha = (n-1)/nl_2$ = 12.77584204, 代入变幅器的频率方程, 求 f_0 然后将 l_3 按步长为0.001求出 Δf, 计算 J_0、J_1 及 N_0、N_1 时可参阅文献[2]。计算结果为变幅器辐射面每磨去0.01mm, 其共振频率上升约2Hz。

4 实测结果和结论

实测结果见表1。

对研磨楔形变幅器辐射面引起共振频率上升这一特定命题作定量研究, 对于设计超声清洗系统的频率跟踪和估算变幅器的寿命是完全必要的。在本文所给的参数条件下, 理论值为每磨去0.01mm, 共振频率上升约为2Hz, 而实测值(4组数据平均值)为1.92Hz/0.01mm。对于不同材料、不同结构的变幅器理论计算应另行处理, 其方法和实验验证与本文所述方法基本相同。

表1

序号	原始共振频率 f_0(Hz)	研磨后(l_3- 0.2mm) 共振频率	$\Delta f/\Delta l$= (Hz/0.01mm)
1	19320	19356	1.8
2	19283	19323	2.0
3	19301	19345	2.2
4	19336	19370	1.7

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声的重要因素, 系统联接刚度过小, 法向载荷过大也容易引发尖叫摩擦噪声。

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