# A modified fast field program model using new admittance formula at elastic sea bottom

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Abstract: As the improved fast field program (FFP) model introduced by West is not applicable to the elastic sub-bottom environment, we propose a modification to the model. Based on the analysis of sound propagation in the elastic medium, we derive the admittance formula at the lowest boundary when the sub-bottom is an elastic half space. With the derived formula, the modified FFP model can be used to calculate the sound field in shallow water with an elastic sub-bottom. Results of two test cases verify the validity of the proposed method. Furthermore, sound propagation in the shallow water with an elastic sub-bottom is studied based on the modified FFP. The results demonstrate that the elastic sub-bottom has much stronger influence on low frequency sound.

Key words: fast field program (FFP); elastic bottom

# 弹性海底边界的声导纳公式及对快速场模型的改进

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**摘要**：West 对基于传播矩阵的快速场算法进行改进，采用了迭代算法，使得基于传播矩阵的快速场算法的计算速度、计算精度以及稳定性有很大改进。但是，迭代算法仅适用于分层液态介质环境，当海底为弹性介质时，迭代算法无法使用。通过对弹性介质中的声传播规律进行研究，推导了弹性半空间边界处的声导纳计算公式。用该公式替代文献[4]给出的边界处声导纳计算公式后，迭代算法可以计算海底为弹性介质时的声场。根据所述方法编制了相应的快速场程序，通过算例比较，说明了该方法的有效性。在典型的浅海环境下，用快速场计算了不同频率的声传播损失曲线。计算结果表明，海底存在切变波时，低频声传播受到的影响较大。因此在考虑低频声传播时，将海底建模为弹性介质是必要的。

**关键词**：快速场；弹性海底

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## 1 INTRODUCTION

Fast Field Program (FFP) is a numerical method to model the acoustic field in horizontally stratified media. The model divides into two parts. First, the depth-separated wave equation is solved at a discrete number of horizontal wavenumbers. Secondly, the wavenumber integration is evaluated numerically. Propagator matrix approach is one of the most widely used methods to numerically solve the depth-separated wave equation in ocean acoustic. The main advantages of the approach are easy im-

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plementation and little memory requirement[1]. However, it encounters the problems of numerical instability, limited accuracy[2], etc. Lee[3], West[4] had given modified formulation of FFP, which is free from the shortcoming mentioned above. The central step in their methods is to calculate the equivalent impedance or admittances for each interface in succession starting from top/bottom interfaces towards the source. Usually, the top/bottom media are assumed to be homogeneous fluid halfspace, for it is convenient to obtain the impedance or admittances at the top/bottom interfaces. However, when the ocean bottom is elastic, 4×4 propagator matrix is indispensable for all layers including the fluid layers. So the formulation of FFP given by Lee[3] or West[4] is unsuitable.

In this paper, the admittance formula at the boundary of the elastic halfspace is deduced. Replacing the admittance formula at bottom interface with our derived formula, the FFP procedure proposed by West can be used to model the acoustic propagation in the ocean with elastic subbottom. Two test cases were carried out, and the results proved the validity of the proposed method. Using the modified Fast Field Program, the sound propagation in the shallow water is studied when the subbottom is elastic. The results show that the elastic subbottom has much more influence on low frequency sound.

## 2 FAST FIELD PROGRAM BASED ON PROPAGATOR MATRIX APPROACH

Fast Field Program (FFP) is essentially a numerical implementation of the integral transform technique for horizontally stratified media. The model could be decomposed into two parts. The first part is to solve the depth-separated equation, and the second is to evaluate the wavenumber integration.

### 2.1 Numerical solution of the depth-separated equation

In a horizontally stratified environment, applying the Fourier-Bessel transform to the Helmholtz equation, the depth-separated equation is obtained as[5]

$$\frac{d^2P}{d^2z}+(k^2-\xi^2)P=-2\delta(z-z_s) \tag{1}$$

In the usual ocean environments, the depth-dependence of $k^2(z)$ is so complex that Eq.(1) can not be solved analytically. Alternatively, the depth-separated wave equation would be solved numerically. Propagator matrix approach is a popular numerical method to solve the depth-separated equation. But, the propagator matrix approach suffers from numerical instability and limited accuracy. West[4] proposed a new procedure to improve its stability and accuracy, in which the central step is to calculate the admittance for each interface in succession starting from the top/bottom interfaces towards the source. The admittance is defined as,

$$Y(z,\xi)=\frac{U(z,\xi)}{P(z,\xi)} \tag{2}$$

where $P(z,\xi)$ and $U(z,\xi)$ are the Fourier-Bessel transforms of the pressure and the normal component of particle velocity respectively. Subdivide the environment into layers within which analytical solutions can be obtained for depth-separated equation. Assume both the top and bottom of the media are homogeneous fluid halfspaces respectively, so the admittance at topmost and bottommost interface ($z_0$ and $z_N$) are obtained conveniently,

$$Y(z_0,\xi)=\frac{i\gamma_a}{\omega\rho_a} \qquad Y(z_N,\xi)=\frac{i\gamma_b}{\omega\rho_b} \tag{3}$$

where $\gamma_a=\sqrt{\xi^2-k_a^2}$, $\gamma_b=\sqrt{\xi^2-k_b^2}$, $k_a$ and $k_b$ are the wavenumber in the top and bottom halfspace, $\rho_a$, $\rho_b$ denote the density respectively, and $\omega$ is the angle frequency. If all the layers are fluid, the propagator matrix is 2×2 in size. Hence, the successive admittance calculation procedure can be derived. Therefore, the admittance at every interface can be calculated in succession starting from top/bottom interface towards the source. At the source layer, integrate Eq. (2) for depth z from $z_s-\delta$ to $z_s+\delta$, and let $\delta\to 0$, we have

$$P(z_s,\xi)=\frac{-\dfrac{2i}{\omega\rho_s}}{Y^+(z_s,\xi)-Y^-(z_s,\xi)} \tag{4}$$

where, $Y^{\mp}(z_s,\xi)=Y(z_s\pm\delta,\xi)$ are the admittance below and above the source.

Due to the fact that the successive pressure calculation procedure can also be derived from the propagator matrix, the pressure can be calculated successively from the source to the detector depth. The procedure of numerical solution to the depth-separated equation is summarized as follows:

(a) determine the admittance at top/bottom interfaces $Y(z_0,\xi)$ $Y(z_N,\xi)$;

(b) calculate the admittance from top interface to the source successively, and determine $Y^-(z_s,\xi)$;

(c) calculate the admittance from bottom interface to the source successively, and determine $Y^+(z_s,\xi)$;

(d) determine the pressure at source depth $P(z_s,\xi)$

(e) calculate the pressure from the source depth to the detector depth successively.

2.2 Wavenumber integration

To determine the acoustic field at a particular receiver range r and depth z, we must numerically evaluate the inverse Fourier-Bessel transform of the solution to the depth-separated wave equation. The following steps are needed to calculate the wavenumber integration numerically:

(a) express Bessel function in terms of the Hankel functions, and then neglect the incoming wave;

(b) replace Hankel function by its asymptotic form;

(c) shift the integration slightly below the real axis to eliminate the aliasing.

(d) truncate the integration interval at the wavenumber $k_{max}$;

(e) approximate the continual integration with discrete summation;

Then the inverse Fourier-Bessel transform can be determined by means of FFT[2],

$$P(r_n,z)=e^{\mu r_n}\frac{\Delta\xi}{\sqrt{2\pi r_n}}e^{i(\xi_{min}r_n-\frac{\pi}{4})}\sum_{m=0}^{M-1}[P(z,\xi_m)\sqrt{\xi_m}\,e^{im\Delta\xi\cdot r_{min}}]e^{imn\Delta\xi\cdot\Delta r} \quad (5)$$

where, $\xi_m=\xi_{min}+m\cdot\Delta\xi$, $r_n=r_{min}+n\cdot\Delta r$.

From the discussion above, we can see that determining the admittance at top/bottom layer is the first step for the modified Fast Field Program. For the cases of fluid top/bottom halfspace, it is convenient to obtain the admittance at top/bottom interfaces. However, obtaining the admittance becomes much difficult in the cases of the elastic ocean bottom.

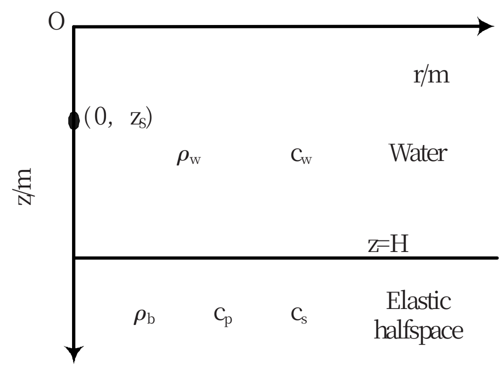


Fig1 Shallow water waveguide with homogeneous elastic bottom

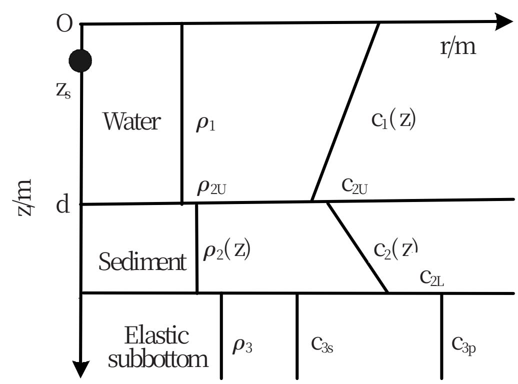


Fig.2 Shallow water waveguide with liquid sediment layer and elastic subbottom

# 3 ADMITTANCE AT THE BOUNDARY OF ELASTIC HALFSPACE

Suppose the ocean bottom is a homogenous elastic halfspace with a compressional speed $c_p$, shear speed $c_s$ and density $\rho_b$, as shown in Figure 1. $c_w$, $\rho_w$ is the density and sound speed in the water respectively. In the elastic media, the scalar displacement potential $\Phi(z,r)$ and the $\theta$ component of the vector displacement potential (in a cylindrical coordinate system) $\Psi(z,r)$ can be denoted as[5],

$$\Phi(z,r)=\int_0^{+\infty} g(z,\xi)J_0(\xi r)\xi d\xi$$
$$\Psi(z,r)=\int_0^{+\infty} f(z,\xi)J_1(\xi r)\xi d\xi \quad (6)$$

$J_v(\cdot)$ is the Bessel function with the order v, $g(z,\xi)$ and $f(z,\xi)$ are the solutions to the following depth-separated wave equations,

$$\frac{d^2g(z,\xi)}{dz^2}+(k^2-\xi^2)g(z,\xi)=0$$
$$\frac{d^2f(z,\xi)}{dz^2}+(\chi^2-\xi^2)f(z,\xi)=0 \quad (7)$$

$k=\omega/c_P$, $\chi=\omega/c_S$, are the wavenumber of compressional wave and shear wave. In the elastic halfspace, the solution to Eq.(7) is,

$$g(z,\xi)=De^{-\beta z} \qquad f(z,\xi)=Me^{-\gamma z} \quad (8)$$

where, D, M are the constant coefficients which would be determined according to boundary conditions, and $\beta=\sqrt{\xi^2-k^2}$, $\gamma=\sqrt{\xi^2-\chi^2}$. According to the relationship between strain, stress as well as particle displacement[6], the tangential stress $T_{zr}(z,\xi)$, normal stress $T_{zz}(z,\xi)$, tangential particle velocity $U_r(z,\xi)$ and normal particle velocity $U_z(z,\xi)$ are,

$$T_{zr}(z,\xi)=\rho\omega^2\frac{2\xi}{\chi^2}[\beta De^{-\beta z}-\sigma Me^{-\gamma z}]$$
$$T_{zz}(z,\xi)=\rho\omega^2\frac{2\xi}{\chi^2}[\sigma De^{-\beta z}-\gamma Me^{-\gamma z}] \quad (9)$$
$$U_z(z,\xi)=-i\omega(-\beta De^{-\beta z}+\xi Me^{-\gamma z})$$
$$U_r(z,\xi)=-i\omega(-\xi De^{-\beta z}+\gamma Me^{-\gamma z}) \quad (10)$$

where, $\sigma=(2\xi^2-\chi^2)/2\xi$. At the interface z=H, the normal stress and particle velocity are continual and the tangential stress is zero. The admittance could be derived at the boundary of the elastic halfspace,

$$Y(H,\xi)=\frac{U_z(H,\xi)}{T_{zz}(H,\xi)}=\frac{-i(-\sigma+\xi)}{\rho\omega^2\frac{2\xi}{\chi^2}(\frac{\sigma^2}{\beta^2}-\gamma)} \quad (11)$$

Substitute Eq.(3) with Eq.(11), the FFP procedure proposed by West can be used to model the acoustic propagation with a elastic bottom.

Although the admittance has no obvious physical sense, we can clearly see the influence of shear wave through the analysis of Eq. (11). Rewrite Eq. (11) as

$$Y(H,\xi)=\frac{-\frac{i\beta}{\rho\omega}}{[(2\frac{\xi^2}{\chi^2}-1)^2+\frac{k}{\chi}4\frac{\xi^2}{\chi^2}\sqrt{1-\frac{\xi^2}{k^2}}\sqrt{1-\frac{\xi^2}{\chi^2}}]} \quad (12)$$

The numerator of Eq.(12) is the admittance formula for fluid halfspace. And the shear wave speed only influences the denominator. While $c_s\to c_p$, or $c_s\to 0$, the denominator approaches 1, and for $c_s\to 0$ it means the shear wave is small, then for $c_s\to c_p$, the shear speed approaches the compressional speed. In a word, the shear has little influence while the shear speed approaches either 0 or compressional speed.

## 4 EXAMPLES

We have deduced the admittance formula at the boundary of the elastic halfspace in Eq.(11). Replacing the admittance formula at bottom interface in Eq.(3) with Eq.(11), the formulation of FFP proposed by West can be used to model acoustic propagation in the ocean with elastic subbottom. In this section, two test cases were carried out. Here, OASES[7] was chosen as the benchmark model to verify the validity of our method. Then, using the modified Fast Field Program, the sound propagation in the shallow water is studied when the subbottom is elastic.

### 4.1 Example 1

A modified Pekeris waveguide is shown in Fig.1 to include the shear in the bottom. It is assumed that the ocean bottom is elastic halfspace with a compressional speed $c_p$=1800m/s, shear speed $c_s$= 600m/s, and density $\rho_b$=1800kg/m³. The ocean depth is assumed to be H=100m. In the water, the sound speed is 1500m/s, and density is 1000 kg/m³. The attenuation is neglected in both water and bottom. The source locates at 95m depth with the frequency 20Hz.

The depth-dependent Green's function and the propagation loss were calculated via the modified FFP. As shown in Fig.3, (a) together with (c) describe the magnitude of the depth-dependent Green's function at 99m and 50m depth respectively, (b) as well as (d) denote the propagation loss at the depth of 99m and 50m as the solid line, propagation loss calculated by OASES is also shown as the dotted line. From the Fig.3 (b), (d), we can see the propagation loss predicted by FFP are consistent well with that via OASES. In general, the peaks of the Green's function correspond to the propagation mode in the waveguide. There are two peaks in Fig.3(c), which indicate two propagation modes in the waveguide. From Fig.3(a), a dramatic shear effect is the existence of an additional peak in the evanescent part of the spectrum ($\xi$>0.0838). This peak corresponds to an additional mode of propagation called Scholte wave in underwater acoustic, which propagates along the interface and decay exponentially as the increase in distance to the interface. So its effect is most pronounced near the water-bottom interface. This wave changes the mode interference pattern as shown in Fig3(b). At the depth of 50m, the Scholte wave has insignificant effect because of the large distance to the interface, which can be seen in Fig.3(c), (d).

4.2 Example 2

Consider the shallow water environment as shown in Fig.2. The environment consists of three layers, water layer, sediment layer and subbottom. The water layer and sediment layer are fluid, and the subbottom is an elastic halfspace. The sound speed profile in the water layer is downward refracting with constant negative gradient. The sound speed at top and bottom are 1500m/s and 1480m/s, respectively. The density of the water is 1000kg/m$^3$. And the depth of the water layer is 100m. The thickness of the sediments is assumed to be 10m. In the sediments, the sound speed is assumed to increase linearly with depth, that is 1580m/s at the top, 1630m/s at the bottom. The density of the sediments is 1600kg/m$^3$. The attenuation in the sediments is 0.4dB/λ. The subbottom is elastic halfspace with a compressional speed $c_p$=1800m/s, shear speed

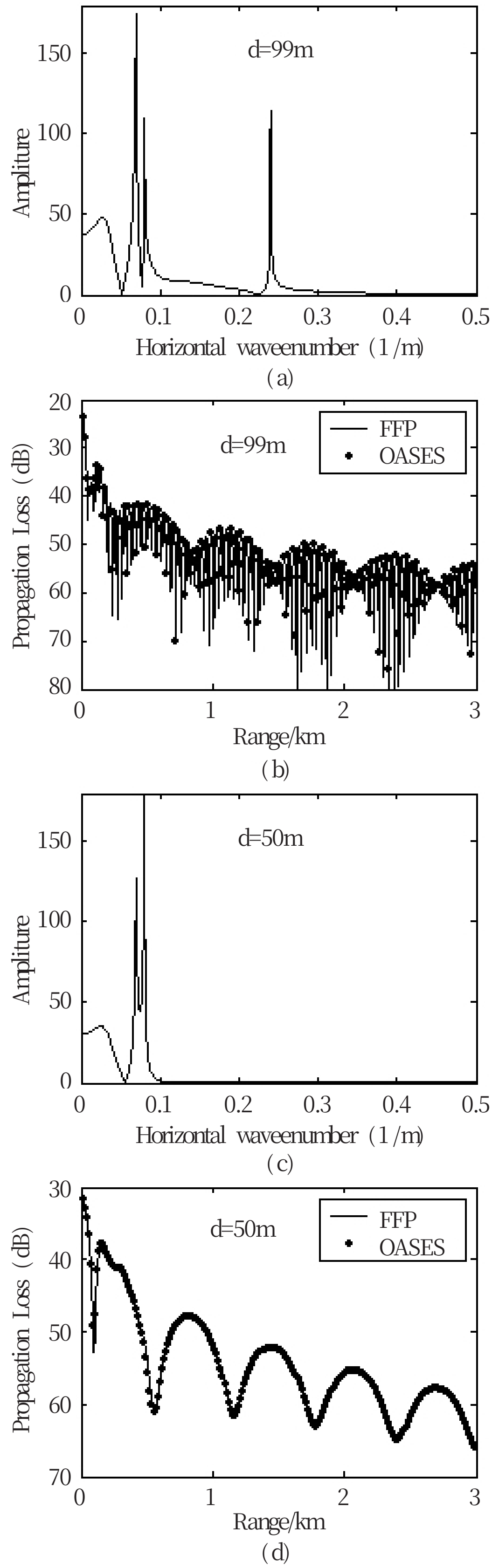


Fig.3 Green's function and Propagation Loss in shallow water waveguide with elastic bottom

$c_s$=600m/s, and density $\rho_b$=2000kg/m$^3$. The compressional wave attenuation is 0.2dB/λ, and shear wave attenuation is 0.3 dB/λ. The source is placed in the water at the depth of 30m, with a frequency of 100Hz. The propagation losses versus range at several depths are calculated with FFP and OASES, as shown in Fig.4 (a) (b) (c) (d). The receiver is located at the depth of 10m, 50m, 99m and 109m respectively. We can see that the propagation loss is calculated by the two models have little difference.

4.3 Example 3

Consider three-layer environment as described in example 2, but the parameters in each layer differ. Here, water is homogeneous with sound speed 1500m/s, and density 1000kg/m$^3$. The water depth is 100m. The thickness of the sediment layer is 5m. And the sediments is uniform fluid with sound speed 1600m/s, density 1600kg/m$^3$ and attenuation 0.2dB/λ. The subbottom is an elastic halfspace, with compressional speed 1800m/s, shear speed 600m/s, compressional wave attenuation 0.3dB/λ, shear wave attenuation 0.4dB/λ and density of 1800kg/m$^3$. The source depth is 20m, and the receiver depth is 50m in the water. We have chosen two source frequencies, 50 and 500Hz, to demonstrate the effects at "low" and "high" frequencies, respectively. Fig.5 (a) and (b) shows the propagation loss for the 50Hz and 500Hz source, separately. The solid line is the propagation loss when the shear vanish in the elastic subbottom, that is the subbottom is fluid. Fig.5 shows that the shear has significant influence on low frequency propagation, but little on high frequency propagation. So it is indispensable to model the subbottom as elastic media while we studying the low frequency acoustic propagation in shallow water.

## 5 CONCLUSION

In this paper, an improvement is presented to the FFP procedure proposed by West. Using the admittance formula of the elastic halfspace, the modified FFP procedure can model the acoustic propagation with elastic subbottom. So the FFP procedure proposed by West can handle more actual ocean environment model. Two test cases were carried out, and the validity of our method was shown. The

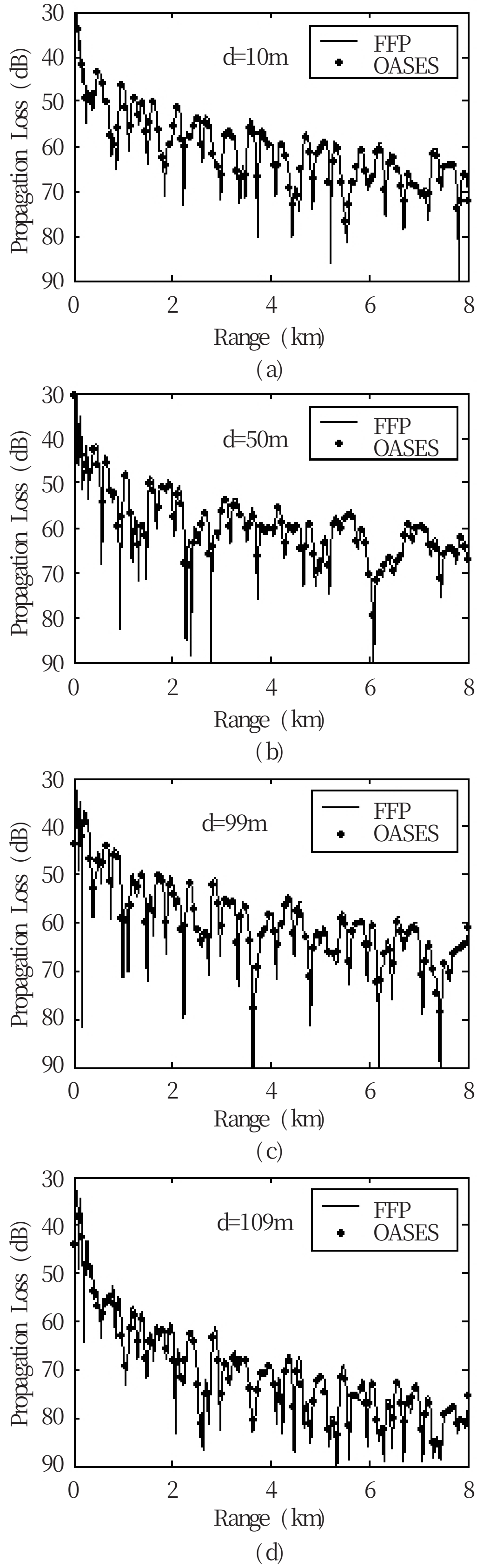


Fig.4 Comparison of transmission Loss calculated with Fast Field Program and OASES

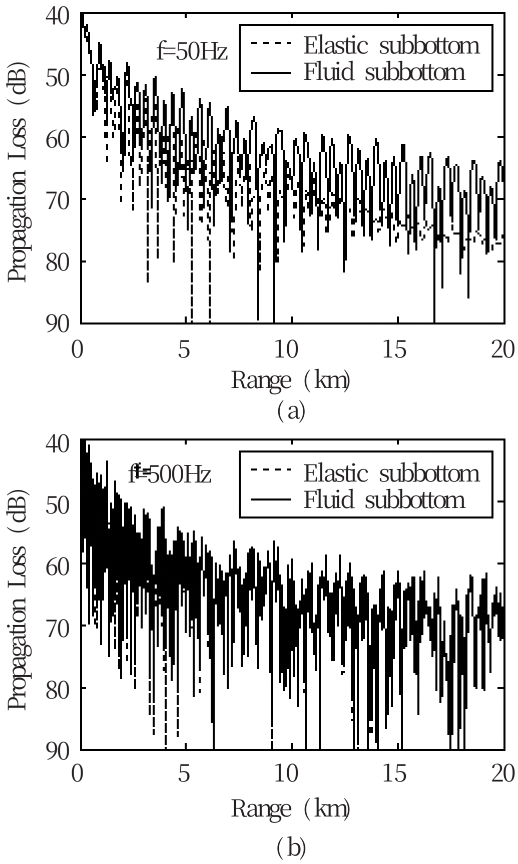


Fig.5 Comparison of the propagation loss for different source frequency when subbottom is elastic

shallow water acoustic propagation was studied with the improved FFP. The results show that it is indispensable to model the subbottom as elastic media when the source frequency is low.

When the subbottom is elastic and its parameters vary with depth, the method presented in this paper no longer applies. For such subbottom, no simple and convenient admittance formula can be deduced at the boundary of the subbottom. So the FFP procedure proposed by West is no longer suitable. It is the future work to model the acoustic propagation with a stratified elastic subbottom, by using the FFP formulation.

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