A novel underwater-target tracking algorithm with experimental study

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Abstract: In view of practical applications of underwater tracking of moving-targets，the rate measure obtained with sonar is a relative radial rate. It cannot be used directly，and the sensor data are color contaminated. A new moving target tracking algorithm with an extended measurement vector（i.e. relative radial rate）and an extended state vector is proposed in this paper to improve steady-state estimation accuracy and convergence rate. A new experimental scheme of tracking multi-target in a water-tank environment is presented. Comparison of performance between the proposed method and the measurement conditioned（MC）approach is given. The results show that there is significant improvement in the tracking capability over MC method.

Key words: relative radial rate；extended measurement and state vector；water-tank experimentation；multi-target signal

极坐标-直角坐标下水下目标跟踪算法及试验研究

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摘要：在实际水下目标跟踪系统中，由声纳获得的速度量测是一个相对径向速度，并且量测噪声是非白的，为此提出了基于扩展量测和扩展状态的混合坐标系下水下目标跟踪算法。该方法把径向速度量测引入观测模型，提高了目标的状态估计精度和收敛时间；引入了扩充向量，克服了实际水下目标观测噪声非白的影响。提出了水下多目标跟踪水池试验方法。该方法应用静态长线阵和多目标信号源在水池实现了多目标和跟踪体之间的相对运动的模拟。水池试验结果表明所提出的方法性能明显优于基于量测转换跟踪方法（如 MC 方法），具有重大工程应用价值。

关键词：相对径向速度；扩展量测、扩展状态向量；水池试验；多目标信号源

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1 INTRODUCTION

In the underwater tracking system，target dynamics are usually modeled in Cartesian coordinates，while the measurements are directly available in the original sensor coordinates，and often expressed in polar coordinates. For tracking in Cartesian coordinates using polar measurements，the measurement conversion method is widely used[1]. The basic idea is to transform the nonlinear measurement model into pseudo-linear form in the Cartesian coordinate，and estimate the bias and covariance of the converted measurement noise，and then use the Kalman filter. As investigated in[1,2]，there are some fundamental flaws in the measurement-conversion algorithms. For convenience，they are listed as follows. Firstly，the error of the converted measurement is state dependent；secondly，its covariance is estimated on the conditions of the current measurement；thirdly，the converted measurement noise sequence

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is not white anymore. However, in the assumptions of the Kalman filter, the measurement noise is independent of the state, and its covariance is unconditional. As pointed out in[1], these fundamental flaws were ignored or overlooked in the past. As a result, they are by no means optimal.

Literature[3,4] shows that the estimated steady-state accuracy and the constringency rate can be improved by using the rate measurement. Aiming at the practical application of tracking underwater-target, the rate measurement obtained by sonar is a relative radial rate, which cannot be used directly, and the sensor data are color contaminated, in accordance with AR(1) process. A new algorithm has been developed in this paper for tracking underwater moving-target in Polar-Cartesian coordinate systems. In comparison with conventional approach, this paper provides three different ways to address such issue namely (i), an extended measurement vector (i.e. radial relative rate measurement) is proposed to improve the estimated accuracy of steady-state and the constringency rate; (ii), an extended state vector is proposed to improve the state estimation even the sensor data are color contaminated And (iii), a mix Polar-Cartesian coordinate system is selected to model the target motion and position measurements from sensor thus avoiding the indescribable problem of measurement noise. A new experimental scheme of tracking multi-targets in water-tank environment was presented. Some experimental results were obtained for different kinds of target trajectories (with the same measurement noise) and the results showed that there was significant improvement in tracking capability over measurement conditioned (MC) approach.

2 EQUATIONS OF TARGET DYNAMIC

The discrete-time equation of target motion can be expressed by a state variable model[5,6]:

$$x(k+1)=\Phi(k+1,k)x(k)+w(k) \quad (1)$$

where

$$x(k)=[x(k),\dot{x}(k),\ddot{x}(k),y(k),\dot{y}(k),\ddot{y}(k)]^T$$

$$\Phi(k+1,k)=\begin{bmatrix} T(k+1,k) & 0_{3\times3} \\ 0_{3\times3} & T(k+1,k) \end{bmatrix} \quad (2)$$

where $T(k+1,k)=\begin{pmatrix} 1 & T & (-1+\alpha T+e^{-\alpha T})/\alpha^2 \\ 0 & 1 & (1-e^{-\alpha T})/\alpha^2 \\ 0 & 0 & e^{-\alpha T} \end{pmatrix}$,

The covariance of $\{W(k)\}$ is,

$$Q(k)=E[w(k)w^T(k)]=2\alpha\sigma_a^2(q_{ij})_{3\times3} \quad (3)$$

where

$$q_{11}=(1-e^{-2\alpha T}+2\alpha T+\frac{2\alpha^3T^3}{3}-2\alpha^2T^2-4\alpha Te^{-\alpha T})/(2\alpha^5)$$

$$q_{12}=(e^{-2\alpha T}+1-2e^{-\alpha T}+2\alpha Te^{-\alpha T}-2\alpha T+\alpha^2T^2)/(2\alpha^4)$$

$$q_{22}=(4e^{-\alpha T}-3-e^{-2\alpha T}+2\alpha T)/(2\alpha^3)$$

3 TRACKING IN POLAR-CARTESIAN COORDINATE WITH ESTEND MEASUREMENT AND STATE VECTOR

In practical application of tracking underwater-targets, we can get the nonlinear sensor measurement r, $\dot{r}$, θ, where r, $\dot{r}$, θ, are measured range, relative range rate and bearing respectively. Converting the Cartesian coordinates measurements into polar yields,

$$\begin{cases} r_k=\sqrt{x_k^2+y_k^2} \\ \dot{r}_k=\dfrac{x_k\dot{x}_k+y_k\dot{y}_k}{\sqrt{x_k^2+y_k^2}} \\ \theta_k=\tan^{-1}(y_k/x_k) \end{cases} \quad (4)$$

These approximations are obtained by taking the first-order terms of Taylor series expansion at $(\hat{x}_{k|k-1},\hat{\dot{x}}_{k|k-1},\hat{y}_{k|k-1},\hat{\dot{y}}_{k|k-1})$

$$r_k=\hat{r}_{k|k-1}+\left[\frac{\partial r_k}{\partial y_k}\right]_{x_k=\hat{x}_{k|k-1}}(x_k-\hat{x}_{k|k-1})+\left[\frac{\partial r_k}{\partial y_k}\right]_{y_k=\hat{y}_{k|k-1}}(y_k-\hat{y}_{k|k-1})$$

$$=\hat{r}_{k|k-1}+\frac{\hat{x}_{k|k-1}}{\hat{r}_{k|k-1}}(x_k-\hat{x}_{k|k-1})+\frac{\hat{y}_{k|k-1}}{\hat{r}_{k|k-1}}(y_k-\hat{y}_{k|k-1})$$

$$=\frac{\hat{x}_{k|k-1}}{\hat{r}_{k|k-1}}x_k+\frac{\hat{y}_{k|k-1}}{\hat{r}_{k|k-1}}y_k \quad (5)$$

where

$$\hat{r}_{k|k-1}=\sqrt{\hat{x}^2_{k|k-1}+\hat{y}^2_{k|k-1}}$$

$$\dot{r}_k=\hat{\dot{r}}_{k|k-1}+\left[\frac{\partial \dot{r}_k}{\partial x_k}\right]_{x_k=\hat{x}_{k|k-1}}(x_k-\hat{x}_{k|k-1})+\left[\frac{\partial \dot{r}_k}{\partial y_k}\right]_{y_k=\hat{y}_{k|k-1}}(y_k-\hat{y}_{k|k-1})$$

$$+\left[\frac{\partial \dot{r}_k}{\partial \dot{x}_k}\right]_{\dot{x}_k=\hat{\dot{x}}_{k|k-1}}(\dot{x}_k-\hat{\dot{x}}_{k|k-1})+\left[\frac{\partial \dot{r}_k}{\partial \dot{y}_k}\right]_{\dot{y}_k=\hat{\dot{y}}_{k|k-1}}(\dot{y}_k-\hat{\dot{y}}_{k|k-1})$$

$$=\left[\frac{\hat{x}_{k|k-1}}{\hat{r}_{k|k-1}}-\frac{\hat{\dot{x}}_{k|k-1}\hat{x}_{k|k-1}(\hat{x}_{k|k-1}+\hat{y}_{k|k-1})}{(\hat{r}_{k|k-1})^{3/2}}\right]x_k+\frac{\hat{x}_{k|k-1}}{\hat{r}_{k|k-1}}\dot{x}_k$$
$$+\left[\frac{\hat{y}_{k|k-1}}{\hat{r}_{k|k-1}}-\frac{\hat{\dot{y}}_{k|k-1}\hat{y}_{k|k-1}(\hat{x}_{k|k-1}+\hat{y}_{k|k-1})}{(\hat{r}_{k|k-1})^{3/2}}\right]y_k+\frac{\hat{y}_{k|k-1}}{\hat{r}_{k|k-1}}\dot{y}_k$$
$$+\left[\frac{(\hat{x}_{k|k-1}+\hat{y}_{k|k-1})(\hat{\dot{x}}_{k|k-1}\hat{x}^2_{k|k-1}+\hat{\dot{y}}_{k|k-1}\hat{y}^2_{k|k-1})}{(\hat{r}_{k|k-1})^{3/2}}-\hat{\dot{r}}_{k|k-1}\right] \quad (6)$$

$$\theta_k=\hat{\theta}_{k|k-1}+\left[\frac{\partial\theta_k}{\partial x_k}\right]_{x_k=\hat{x}_{k|k-1}}(x_k-\hat{x}_{k|k-1})$$
$$+\left[\frac{\partial\theta_k}{\partial y_k}\right]_{y_k=\hat{y}_{k|k-1}}(y_k-\hat{y}_{k|k-1})$$
$$=\tan^{-1}\frac{\hat{y}_{k|k-1}}{\hat{x}_{k|k-1}}-x_k\frac{\hat{y}_{k|k-1}}{\hat{r}^2_{k|k-1}}+y_k\frac{\hat{x}_{k|k-1}}{\hat{r}^2_{k|k-1}} \quad (7)$$

Define an extended measurement vector $Z(k)=[r_k, \dot{r}_k, \theta_k]'$, then the linear measurement model can be observed,

$$\boldsymbol{Z}(k)=\boldsymbol{H}(k)(\bar{\boldsymbol{X}})\boldsymbol{X}(k)+D(\bar{\boldsymbol{X}})+\boldsymbol{V}(k) \quad (8)$$

where,

$$\boldsymbol{D}(\bar{X})=\begin{bmatrix}0\\ (\hat{x}_{k|k-1}+\hat{y}_{k|k-1}+\hat{z}_{k|k-1})(\hat{\dot{x}}_{k|k-1}\hat{x}^2_{k|k-1}+\hat{\dot{y}}_{k|k-1}\hat{y}^2_{k|k-1}\\ +\hat{\dot{z}}_{k|k-1}\hat{z}^2_{k|k-1})/(\hat{r}_{k|k-1})^{3/2}-\hat{\dot{r}}_{k|k-1}\\ \tan^{-1}(\hat{y}_{k|k-1}/\hat{x}_{k|k-1})\end{bmatrix} \quad (9)$$

$$\boldsymbol{H}(\bar{X})=\begin{bmatrix}h_1 & 0 & 0 & h_2 & 0 & 0\\ h_3 & h_4 & 0 & h_5 & h_6 & 0\\ h_1 & 0 & 0 & h_8 & 0 & 0\end{bmatrix} \quad (10)$$

where

$$h_1=\hat{x}_{k+1|k}/\hat{r}_{k+1|k},\ h_2=\hat{y}_{k+1|k}/\hat{r}_{k+1|k}$$

$$h_3=\frac{\hat{x}_{k|k-1}}{\hat{r}_{k|k-1}}-\frac{\hat{\dot{x}}_{k|k-1}\hat{x}_{k|k-1}(\hat{x}_{k|k-1}+\hat{y}_{k|k-1})}{(\hat{r}_{k|k-1})^{3/2}},\ h_4=\frac{\hat{x}_{k|k-1}}{\hat{r}_{k|k-1}}$$

$$h_5=\frac{\hat{y}_{k|k-1}}{\hat{r}_{k|k-1}}-\frac{\hat{\dot{y}}_{k|k-1}\hat{y}_{k|k-1}(\hat{x}_{k|k-1}+\hat{y}_{k|k-1})}{(\hat{r}_{k|k-1})^{3/2}},\ h_6=\frac{\hat{y}_{k|k-1}}{\hat{r}_{k|k-1}}$$

$$h_7=-\hat{y}_{k+1|k}/\hat{r}^2_{k+1|k},\ h_8=\hat{x}_{k+1|k}/\hat{r}^2_{k+1|k}$$

The noise components in $V(k)$ are commonly assumed to be statistically independent and white. Unfortunately, according to the analysis for underwater-target and real test data, we find that the adequate model of them is AR(1) process, as

$$\boldsymbol{V}(k)=\beta\boldsymbol{V}(k-1)+\xi(k) \quad (11)$$

where $\beta=diag(\beta_1,\ \beta_2,\ \beta_3,)$ is a constant vector. $\xi(k)$ is 3×1 dimensional zero-mean white noise with the covariance $E[\xi(k)\xi^T(j)]=\boldsymbol{R}(k)\xi_{kj}\quad k,j=1,2,3,\cdots$

Define an extended state vector by augmenting $V(k-1)$ to $X(k)$, i.e.

$$X^*(k)=\begin{bmatrix}X(k)\\ V(k-1)\end{bmatrix} \quad (12)$$

The model of target motion and observation can now be written as follows,

$$\boldsymbol{x}^*(k+1)=\boldsymbol{\Phi}^*(k+1,k)\boldsymbol{x}^*(k)+\boldsymbol{w}^*(k)$$
$$\boldsymbol{Z}(k)=\boldsymbol{H}^*(\bar{X})X^*(k)+\xi(k) \quad (13)$$

where $\boldsymbol{\Phi}^*(k+1,k)=\begin{bmatrix}\boldsymbol{\Phi}(k+1,k) & 0\\ 0 & \beta\end{bmatrix}, \boldsymbol{w}^*(k)=\begin{bmatrix}\boldsymbol{w}(k)\\ \xi(k)\end{bmatrix}$, $\boldsymbol{H}^*(\bar{X})=[\boldsymbol{H}(\bar{X})\ \beta]$.

The mean and covariance of sequences $\{\boldsymbol{w}^*(k)\}$ and $\{\xi(k)\}$ are

$$E[\xi(k)]=0, E[\xi(k)\xi^{\mathrm{T}}(j)]=\boldsymbol{R}(k)\xi_{kj}$$
$$E[\boldsymbol{W}(k)]=0, E[\boldsymbol{W}(k)\boldsymbol{W}^{\mathrm{T}}(j)]=\boldsymbol{Q}(k)\xi_{kj}$$
$$E[\boldsymbol{W}^*(k)]=0, E[\boldsymbol{W}^*(k)\boldsymbol{W}^{*\mathrm{T}}(j)]=\boldsymbol{Q}^*(k)\xi_{kj}$$
$$\boldsymbol{Q}^*(k)=\begin{bmatrix}\boldsymbol{Q}(k) & 0\\ 0 & \boldsymbol{R}(k)\end{bmatrix}\quad k,j=1,2,3,\cdots \quad (14)$$

The flow-chart of proposed tracking filter in mix coordinates with extend measurement and state vector is summarized as follow,

$$\hat{\boldsymbol{X}}^*(k+1/k)=\boldsymbol{\Phi}^*(k)\hat{\boldsymbol{X}}(k/k) \quad (15)$$
$$\boldsymbol{P}(k+1/k)=\boldsymbol{\Phi}^*(k)\boldsymbol{P}(k/k)\boldsymbol{\Phi}^{*T}(k)+\boldsymbol{Q}^*(k) \quad (16)$$
$$\hat{Z}(k+1/k)=\boldsymbol{H}^*(k+1)\hat{X}(k+1/k)+D(\bar{X}) \quad (17)$$
$$\boldsymbol{S}(k+1)=\boldsymbol{H}(k+1)\boldsymbol{P}(k+1/k)\boldsymbol{H}(k+1)+\boldsymbol{R}(k+1) \quad (18)$$
$$\boldsymbol{K}(k+1)=\boldsymbol{P}(k+1/k)\boldsymbol{H}^T(k+1)\boldsymbol{S}^{-1}(k+1) \quad (19)$$
$$\hat{\boldsymbol{X}}^*(k+1/k+1)$$
$$=\hat{\boldsymbol{X}}^*(k+1/k)+\boldsymbol{K}(k+1)(\boldsymbol{Z}(k+1)-\hat{\boldsymbol{Z}}(k+1/k)) \quad (20)$$
$$\boldsymbol{P}(k+1/k+1)$$
$$=\boldsymbol{P}(k+1/k)-\boldsymbol{K}(k+1)\boldsymbol{S}(k+1)\boldsymbol{K}^T(k+1) \quad (21)$$

where $\boldsymbol{H}(k+1)$, $D(\bar{X})$ can be gotten by(9),(10).

4 EXPERIMENTATION AND PERFORMANCE COMPARISON

We propose a new experimental scheme of tracking multi-target in water-tank environment, which simulates the relative movements between the targets and the tracker with a static long line arrays and multi-target signal. The whole experimental system can be illustrated in Figure 1, where the multi-target signal generates signals with different delays and orientation. The transmitted-array is 16m long, and the interval of array elements is changeable, which simulate the multi-targets with different orientation.

To simulate the ocean environment, the dimensions of water-tank have been adopted as 20m×8m×7m. The receiving array is a uniform linear array with 14 array elements. The DSP system fulfills A/D transform of many channels and the multi-target locating and tracking algorithms. The main steps of test are list as follow:

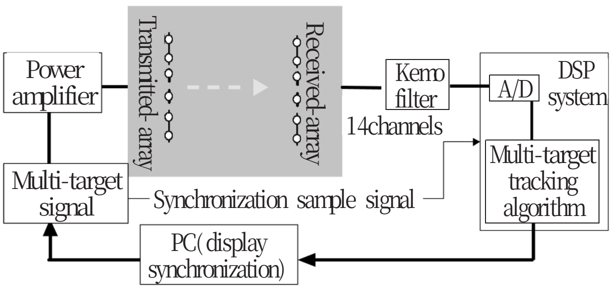

Fig.1 The water-tank experimentation system

(1) Calculate the targets' orientation and echo delay based on the relative position between the targets and the tracker, and the generated echo.

(2) Multi-target signal automatic choose the emitting-array and emit the signals.

(3) The incept-array receives echo. The DSP system estimates the multi-targets' parameters and fulfills the multi-target tracking.

(4) If it hits the target or voyage over, then end, otherwise go back to step 1.

We choose the measurement conditioned (MC) approach, which is the best among other top choices in the class of the measurement conversion method. They were referred to as unbiased and debaised methods respectively in[8,9,10]. The comparison results of the same scenario are presented as follows.

Considering a two-dimensional Cartesian space with a single sensor located at the origin. Target sampling period is 2 sec. The original position of target is (x(0)=100m, y(0)=100m), the rate (v=10kts), and the bearing (α=45°). The sensor's independent measurement errors give standard deviations $\sigma_r=0.03^* r(m)$, $\sigma_r=17.5^*$ (millirad) and $\sigma_{\dot{r}}=0.05^* \dot{r}(m/s)$.

Table 1 Mean and standard deviation compare by three different methods

	Method	Proposed Method	Debaised MC	Unbaised MC
Mean	Position of X (m)	2.5337	6.0766	8.0716
	Position of Y (m)	3.9174	6.2861	8.0161
	Velocity of X (m/s)	0.0498	-0.0601	-0.1601
	Velocity of Y (m/s)	0.0533	-0.0630	0.1230
Standard deviation	Position of X (m)	3.6551	13.9007	16.127
	Position of Y (m)	4.0884	14.0435	17.015
	Velocity of X (m/s)	0.6485	0.9579	1.1079
	Velocity of Y (m/s)	0.6853	0.9611	1.2011

The results of experiments are presented in Fig. 2 and Table 1 for the performance comparison the algorithm presented in this paper to the MC method. The results indicate that the proposed tracking algorithm gives significant improvement in tracking capability over the MC methods.

5 CONCLUSION

A practical application in tracking of moving target, with extended measurement vector and state vector recursive filter was presented for linear dynamics with nonlinear measurements. This filter is free of the fundamental flaws of the measurement-conversion method.

A new experimental scheme of tracking multi-target in water-tank environment was presented. The experimental-based comparison of the proposed method with measurement-conversion techniques (MC method) demonstrates the following. In terms of estimated errors and filter credibility, the proposed method performs significantly over the MC method in all cases tested; in particular, the proposed method is always almost perfectly credible in the sense that the actual estimated errors are consistent with the filter's self-assessment. Moreover, the good performance is achieved without increase of the computational complexity.

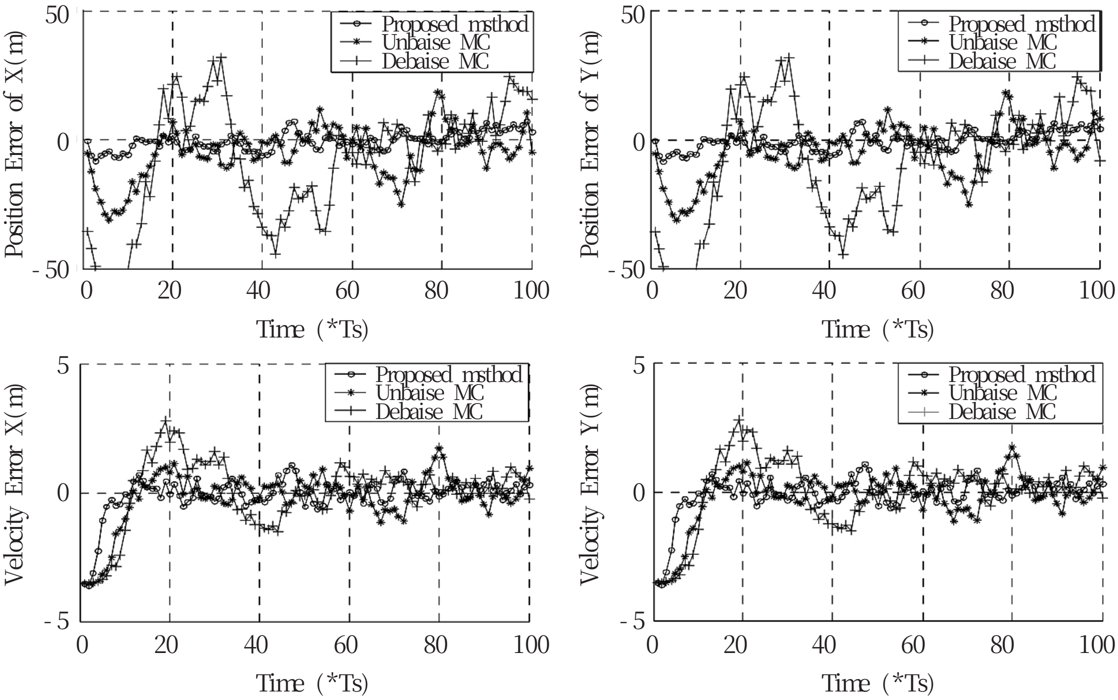

Fig.2 Performance compare by three different methods

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