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Effect of spatial correlation on underwater acoustic MIMO capacity

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Abstract: The effect of channel parameters such as angle spread, angle of arrival, and array element spacing on the underwater acoustic MIMO capacity is studied with a three-dimensional multipath correlation model using a vertical transmitting and receiving array. In a MIMO system with static transmitting and receiving arrays, regardless of the movement of scatters in the underwater acoustic channel, the capacity is dependent on the spatial correlation among array element signals at the transmitter as well as at the receiver. If correlation between elements of the channel matrix is weak, increasing the number of array elements will greatly increase the MIMO capacity. In a channel of strong correlation, increasing the number of array elements will cause the MIMO capacity to approach saturation.

Key words: MIMO; underwater acoustics; spatial correlation; channel capacity

空间相关性对水声多输入多输出系统信道容量的影响

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摘要：通过三维多径相关模型，以垂直收发阵为例研究了信道参数如声线角度扩展范围、到达方向和收发阵元间距对水声 MIMO 信道容量的影响。如果考虑静止的收发阵，且不考虑信道中散射体的运动时，MIMO 系统的信道容量由收发阵元信号的空间相关性决定，当空间相关性较小时，增加收发阵元数可以带来较大的信道容量增益，而当空间相关性较大时，随着收发阵元数的增加信道容量将趋于饱和。

关键词：多输入多输出；水声；空间相关；信道容量

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1 INTRODUCTION

The MIMO technology using multiple array elements at both the tansmiter and the receiver has high potential in improving the spectral efficiency of densely scattered underwater acoustic channels, but so far few researches about the underwater acoustic MIMO capacity have been conducted. Researches[1-3] have shown that, in a Rayleigh flat-fading environment, a MIMO wireless communication link has a theoretical capacity that increases linearly with smaller number of transmiting and receiving array elements, provided that the propagation coefficients among all pairs of transmitting and receiving elements are statistically independent. As such, the MIMO technology has drawn wide interest in the wireless communication field over the past two decades. However, in some practical environment,

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there are some spatial correlations between the propagation coefficients caused by the poor scattering environment or insufficient spacing between the array elements, which affect the MIMO capacity of these practical channels. The effect of transmitting power and signal frequency on the MIMO capacity of a typical shallow water channel was studied through field measurements[4]. However, results obtained in this manner are only applicable to a particular environment and array configuration. Furthermore, it is difficult to study the effect of the environment parameters on MIMO capacity. The MIMO capacity can also be studied through abstract scattering models[5], from which the essential characteristic of the environment can be illuminated, and its effect on the MIMO capacity can easily be studied. The MIMO capacities can be derived when there are correlations in either of the transmitter or the receiver. The effect of channel parameters on MIMO capacity using the scattering model of ‘one-ring’ can also be studied through Monte Carlo Simulation. The dependence of the scaled MIMO capacity with the number of array elements on channel correlation can be studied with an assumption based on some kinds of correlation models[6]. The characteristic function of MIMO capacity was derived when the channel is correlated at either the transmitter or the receiver in a Rayleigh flat-fading channel[7].

The “one-ring” or “two-ring” scattering models are generally adopted in the wireless radio communication systems. The application of “one-ring” model is appropriate when the base station is elevated and unobstructed by local scatters, and likewise, the “two-ring” model is appropriate when both of the transmitter and receiver are surrounded by objects. In these models, the scatters are generally assumed to be distributed on the circles around the transmitting or receiving arrays. However, in the underwater acoustic communication channels, the transmitted and received acoustic rays are spread both vertically and horizontally. This paper studies the effects of various channel parameters on the underwater MIMO capacity through a three dimensional multi-path spatial correlation model.

2 SPATIAL CORRELATION MODEL

In a MIMO communicaiton system which consists of n_R receiver elements and n_T transmitter elements, and there are multi paths between the transmitter and the receiver, the received signal of the ith, $i=\{1,...,n_R\}$ receiving element from the jth, $j=\{1,...,n_T\}$ transmitting element can be expressed as

$$h_{ij}(t)=\sum_{n=1}^{N} A_n \exp(iw(t-\tau_{n,i,j})) \tag{1}$$

where i is an imaginary unit, A_n is the amplitude gain of the nth path, N is the number of the multipath, and $\tau_{n,i,j}$ is the path delay associated with the propagation from the jth transmitting element to the ith receiving element along the nth path. Given the statistical properties of the channel, if N is sufficiently large, the central limit theorem states that $h_{i,j}(t)$ is a zero-mean complex Gaussian-random process. Therefore, the envelope of $|h_{i,j}(t)|$ is a Rayleighfading process.

The correlation between the two links, T_i-R_j and $T_{i'}$-$R_{j'}$, when normalized by the received signal field variance is defined as

$$\rho_{ij,i'j'}(t)=E[h_{ij}(t)h_{i'j'}^{*}(t)]/\sqrt{\Omega_{ij}\Omega_{i'j'}} \tag{2}$$

where * is the complex conjugate, $\mathbf{\Omega}_{ij}=E[|h_{ij}(t)|^2]$, substitute Equ.(1) into Equ.(2) and simplify the result,

$$\rho_{ij,i'j'}(t)=\rho_{ij,i'j'}=E[\sum_{n=1}^{N}\exp(iw(\tau_{n,i',j'}-\tau_{n,i,j}))] \tag{3}$$

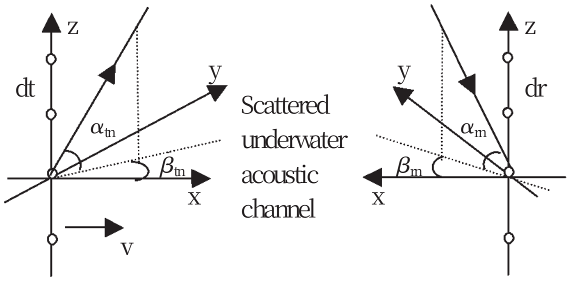


Fig.1 Configuration of the underwater acoustic MIMO communication system(illustrated by a vertical transmitting and a vertical receiving array)

Considering plane waves for the vertical transmit array and vertical receive array, shown in Fig.1, we have

$$\tau_{n,i',j'}-\tau_{n,i,j}=(d_r\sin\alpha_m(i'-i)+d_t\sin\alpha_m(j'-j))/c \quad (4)$$

For the horizontal transmitting and receiving array along the axis of x,

$$\tau_{n,i',j'}-\tau_{n,i,j}=(d_r\cos\alpha_m\cos\beta_m(i'-i)+ d_t\cos\alpha_{tn}\cos\beta_{tn}(j'-j))/c \quad (5)$$

For the horizontal transmitting and receiving array along the axis of y,

$$\tau_{n,i',j'}-\tau_{n,i,j}=(d_r\cos\alpha_m\sin\beta_m(i'-i)+ d_t\cos\alpha_{tn}\sin\beta_{tn}(j'-j))/c \quad (6)$$

where dt and dr are the transmitting and receiving element spacing, α_{tn}, β_{tn} are the elevation angle and azimuth angle of the nth path at transmitter; α_{rn}、β_{rn} are the elevation angle and azimuth angle of the nth path at the receiver respectively, c is the underwater sound speed, in this paper, it is assumed to be constant i. e. c=1500(m/s).

For other array configurations, $\tau_{n,i',j'}-\tau_{n,i,j}$ would have similar expressions. The effect of spatial correlation on the underwater acoustic MIMO capacity can be studied through the vertical transmitting array and vertical receiving array. Substitute Equ.(4) into Equ.(3),

$$\rho_{ij,i',j'}=\langle\exp(iw((i'-i)d_r\sin\alpha_r+ (j'-j)d_t\sin\alpha_t)/c\rangle_{\alpha_t,\alpha_r} \quad (7)$$

where <n> means statistical averaging. Equ.(7) shows that for the vertical transmitting and receiving arrays, the spatial correlation function depends on the spacing of transmitting and receiving elements; the elevation angle spread of the acoustic rays at transmitter and receiver; in this article, they are assumed to be uniformly distributed in $[\Theta-\Delta, \Theta+\Delta]$, where Θ is the angle of arrival and Δ is the angle spread. The effects of these parameters on underwater acoustic MIMO capacity can be studied in such a manner.

3 MIMO CAPACITY

For a narrow-band single-user communication system with n_T transmitter and n_R receiver omni-directional elements, the relationship between the transmitted signal vector and the received signal vector can be expressed

$$y(t)=H(t)*x(t)+n(t) \quad (8)$$

where x(t) is the $n_T\times1$ transmitted signal vector, y(t) is the $n_R\times1$ received signal vector, n(t) is the $n_R\times1$ additive white Gaussian noise vector, and H(t) is the $n_R\times n_T$ channel propagation matrix of complex path gains $h_{ij}(t)$ between transmitter element j and receiver element i. The elements of noise vector are assumed to be independent and identically distributed (i.i.d) complex Gaussian-random variables with variance N_o.

Considering a narrow band flat frequency response communication system, Eq.(8) can be written as,

$$y=Hx+n \quad (9)$$

When the channel state information is known at the receiver but not known at the transmitter, the total transmitted power are generally equally alloc-ated to every transmitter element. In this case, the instant MIMO capacity corresponding to each reali-zation of H is expressed as[1-3],

$$C=\log_2\det(I_{n_R}+(SNR/n_T)HH^+)\ \text{bps/Hz} \quad (10)$$

where H^+ means the conjugated transpose of H, det(·) is the matrix determinant, I_{n_R} is the $n_R\times n_R$ identity matrix, and SNR is the average signal-to-noise ratio (SNR) at each receiver element.

For a random channel matrix H, its capacity C is also a random variable. The Complementary Cumulative Distribution Function (CCDF) is generally used to give a complete description of the random channel capacity. As mentioned earlier, the characteristic function of C is derived in the research[7] where the channel is correlated either at the transmission end or the receiver end, from which the CCDF could be generated. Fig.2 describes the changing of MIMO capacity with the receive correlation when the transmit correlation is kept unchanging. The simulation parameters in Fig.2 are $\alpha_t\in[-5°,5°]$, $\alpha_r\in[-5°,5°]$, $d_r=1\lambda$, $n_T=n_R=6$, SNR=10dB. Fig.2 shows that the result of MIMO capacity in the above research[7] is accurate when there is no correlation or the correlation coefficient is small in the transmission end. When both of the transmission and receiving ends have high correlations, it is difficult to get the distribution function of C, so we use Monte Carlo simulation to get the CCDF of C and study the effect of channel parameters on MIMO capacity.

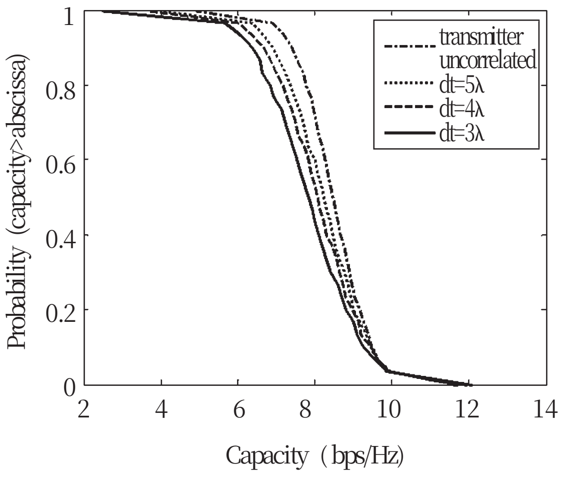


Fig.2 The effect of transmitter correlation on CCDF

4 SIMULAITON RESULTS

4.1 Generation of the correlated variables

Generation of M Gaussian variables with an arbitrary correlation can be achieved by decomposing the desired M×M covariance matrix, $R=GG^+$, where G^+ is the conjugated transpose of G, then multiplying M independent Gaussian variables by G[8].

4.2 Simulation results for CCDF and brief analysis

Fig.3, 5 and 7 describe the dependence of spatial coherence on the parameters of transmitting and receiving element spacing, angle spread and AOA respectively, and illustrated by $|\rho_{21,32}|$. In these figures, two groups of simulation parameters have been chosen, one group has a small correlation and the other has a great correlation, Fig.4, 6 and 8 describe the effects of these parameters on the MIMO capacity with $n_R=n_T=3$, SNR=10dB, in these figures, there are two cases, named by (a) and (b), corresponding to the two groups of parameter setting in Fig.3, 5 and 7. The detailed parameter settings are illustrated in Table.1.

Table 1 Parameter setting of Fig.3~Fig.8

	Parameter
Fig.4	(a) $\alpha_t \in [-5^\circ, 5^\circ]$, $\alpha_r \in [-5^\circ, 5^\circ]$, AOA=0°
	(b) $\alpha_t \in [-5^\circ, 5^\circ]$, $\alpha_r \in [-20^\circ, 20^\circ]$, AOA=0°
Fig.6	(a) dt=dr=1λ, AOA=0°
	(b) dt=dr=5λ, AOA=0°
Fig.8	(a) $\alpha_t \in [-5^\circ, 5^\circ]$, $\alpha_r \in [-5^\circ, 5^\circ]$, dt=dr=1λ,
	(b) $\alpha_t \in [-20^\circ, 20^\circ]$, $\alpha_r \in [-20^\circ, 20^\circ]$, dt=dr=5λ

Fig.4 and 6 show that both the element spacing and angle distribution have great effects on MIMO capacity. From Fig.4, we can see that in a poor scattering environment, to get huge MIMO capacity needs a large element spacing, and that getting a MIMO capacity equal to that of the completely independent channel, the transmitting and re-ceiving element spacing in Fig.4(a) is 8λ, but in Fig.4(b) is only 2λ, which is consistent with the correlation in Fig.3. Similar results can also be obtained in Fig.5 and 6, when the element spacing is not sufficiently large, dense scattering environment is needed to get a large MIMO capacity gains.

Fig.8 illustrates that with large element spacing and angle spread, the AOA will have a great effect on MIMO capacity, as shown in Fig.8(b), otherwise, the effect of AOA on MIMO capacity is very small, shown in Fig.8(a), and these results are also corresponded to the spatial correlation in Fig.7.

Fig.9 illustrates the dependence of MIMO capacity on the number of transmitting and receiving elements with two different correlation conditions, where the parameters, SNR=10dB, and in Fig.9(a) $\alpha_t \in [-20^\circ, 20^\circ]$, $\alpha_r \in [-20^\circ, 20^\circ]$, dt=dr=5λ, AOA=0°, in Fig.9(b) $\alpha_t \in [-2^\circ, 2^\circ]$, $\alpha_r \in [-2^\circ, 2^\circ]$, dt=dr=0.25λ, AOA=0°. Fig.9(a) and (b) represent two cases of the completely independent and completely correlated environments. Fig.9 shows that large MIMO capacity gains can be obtained by using multiple array elements when the correlation in both the transmitting and receiving ends are small, but when there are great correlations, the MIMO capacity gains from the increasing of element number is very limited.

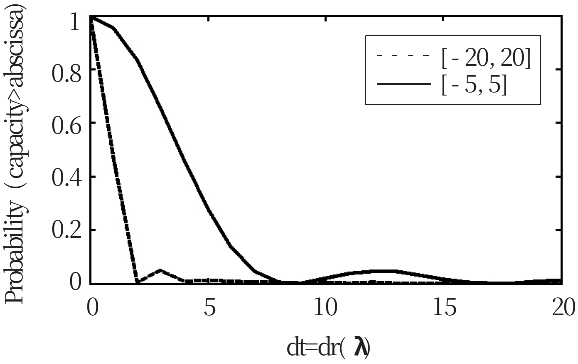
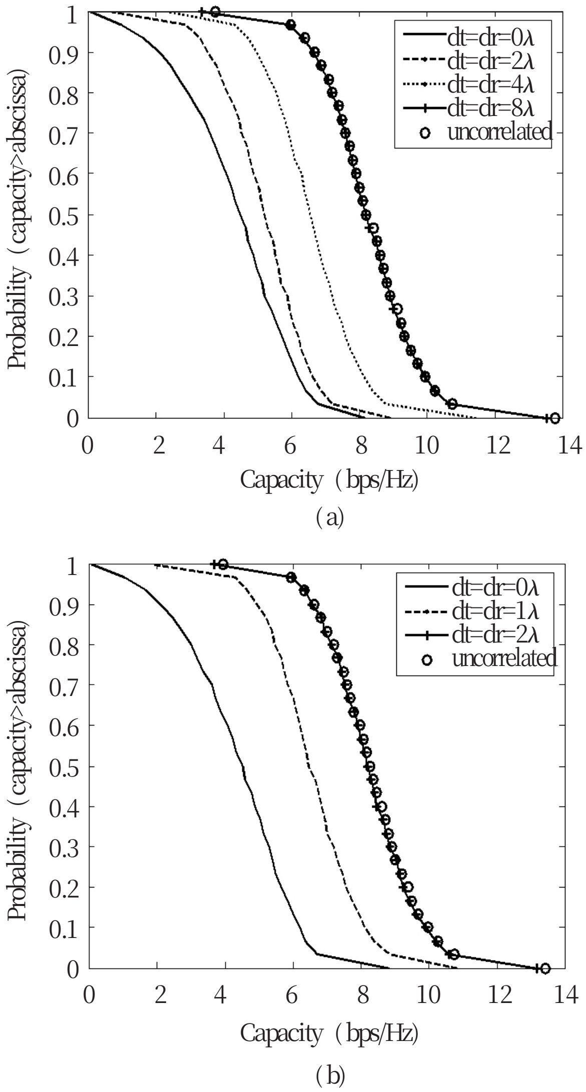


Fig.3 Dependence of $|\boldsymbol{\rho}_{1,32}|$ on element spacing



(a)

(b)

Fig.4 CCDF dependence on array element spacing

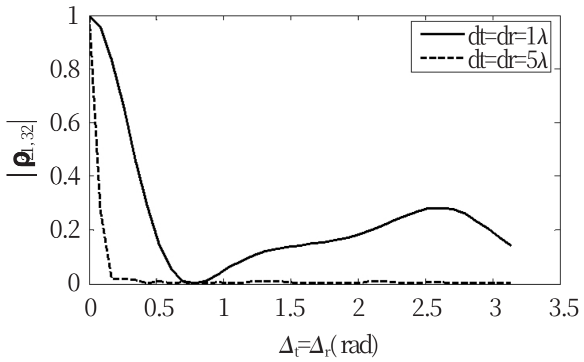
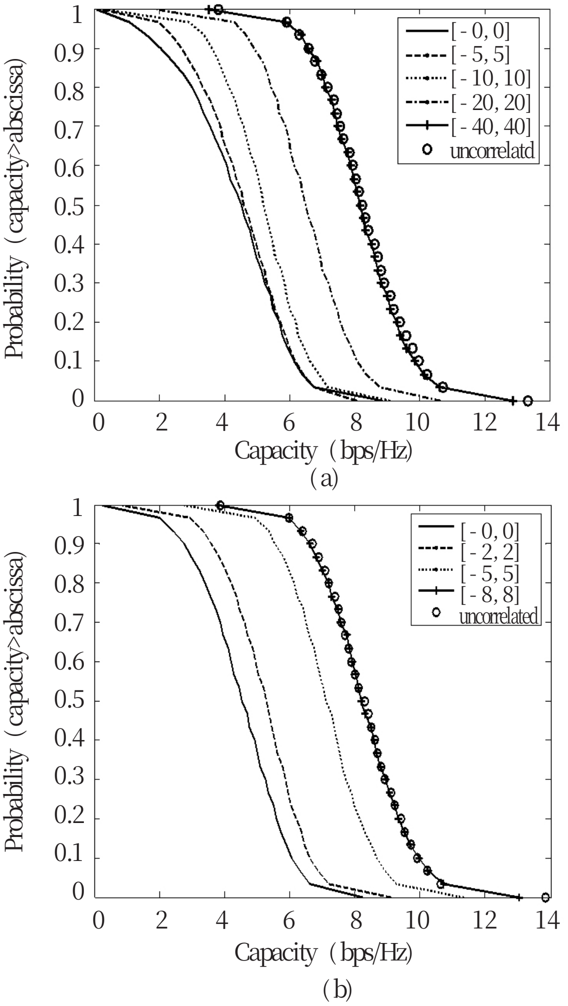


Fig.5 Dependence of $|\boldsymbol{\rho}_{1,32}|$ on angle spread



(a)

(b)

Fig.6 Dependence of CCDF on angle spread

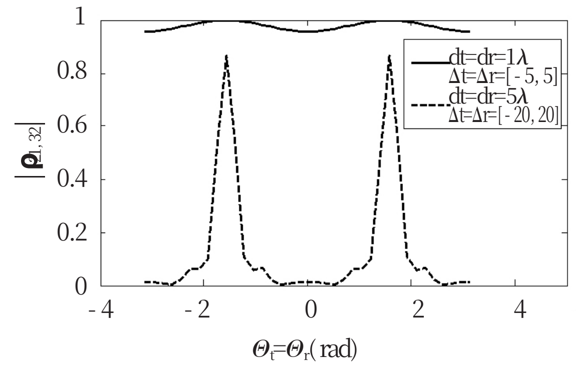
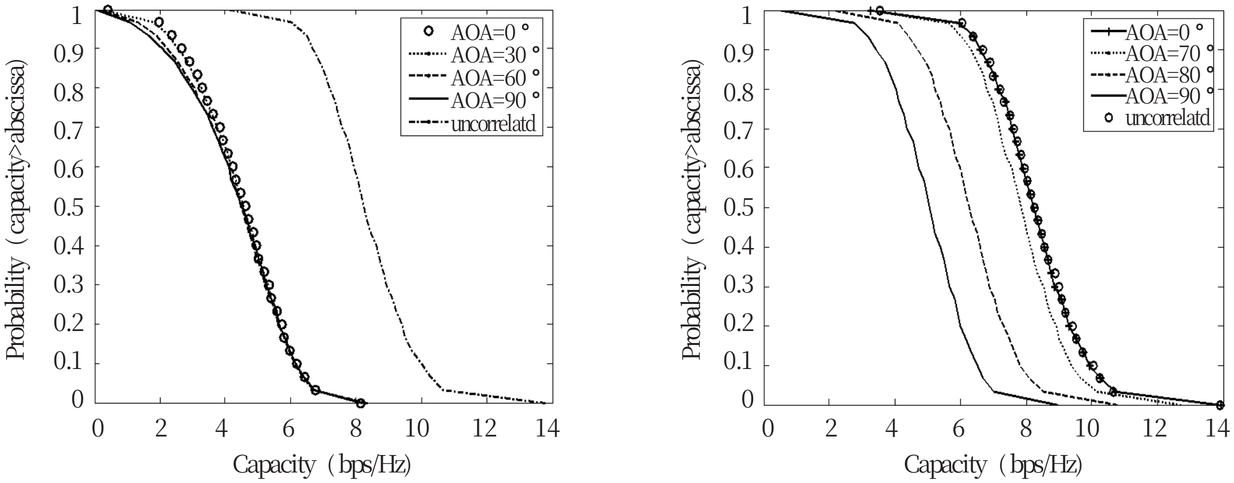
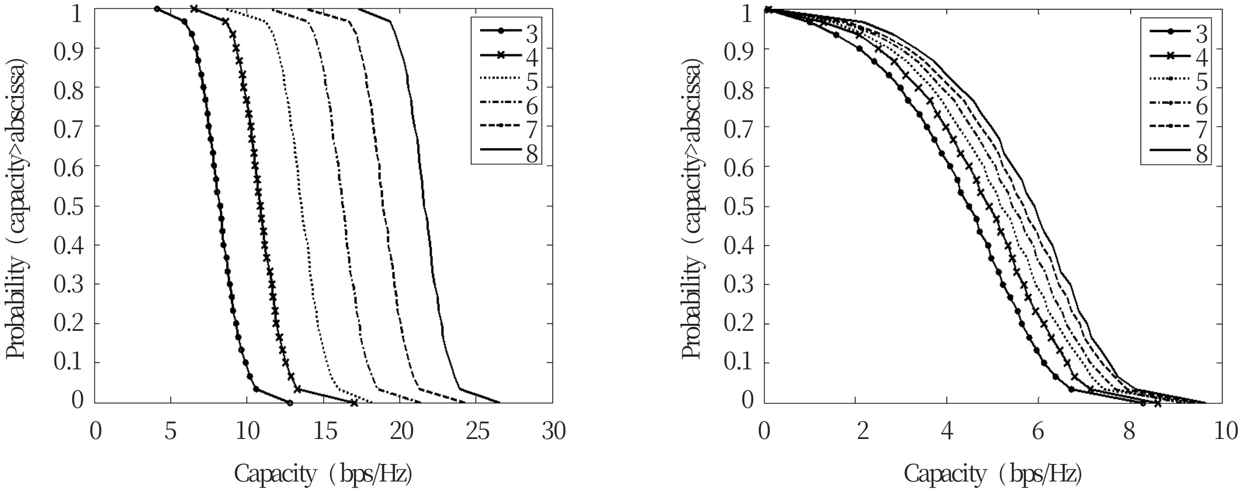


Fig.7 Dependence of $|\boldsymbol{\rho}_{1,32}|$ on AOA



(a) (b)

Fig.8 Dependence of CCDF on AOA



(a) (b)

Fig.9 Dependence of CCDF on array element number

5 CONCLUSION

Parameters of angle spread, angle of arrival, and element spacing decide the correlations between the MIMO channel coefficients, and thus affect the channel capacity because the angle spread and element spacing have similar effects on spatial correlations. Therefore, in the environment of a small angle spread, increasing transmitting and receiving element spacing can also achieve high MIMO capacity gains. Furthermore, for a static MIMO system, the channel capacity gains are decided by the spatial correlations among the channel coefficients. As such, in a system with small correlations, MIMO capacity will increase with the number of elements linearly, but in a great correlation environment, increasing the number of element will not achieve a large MIMO capacity gain.

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简 讯

2006年全国声学学术会议在福建厦门召开

在福建厦门组织召开的2006年全国声学学术会议，共录取论文356篇。会议特别邀请应崇福院士、沈钧贤研究员、李明轩研究员分别就大家十分关心的问题作了大会报告。论文集中刊登论文280篇，包括综述报告3篇，物理声学13篇，水声学62篇，超声学、量子声学与声学物理效应56篇，噪声、噪声效应及其控制34篇，结构与建筑声学14篇，语言声学与语音通讯14篇，生理与心理声学11篇，生物声学1篇，音乐声学3篇，声学测量、信号处理与分析的方法、仪器55篇，声学换能器14篇。这些论文充分反映了我国声学工作者在声学研究和声学技术等领域中所取得的最新成就。

这次会议给各位会议代表提供了一个学术交流的平台，会议代表们来自全国各地，工作岗位各有不同，看问题的角度也不会相同。通过学术交流，宏观地探讨了我国声学科学的现状，展望了未来的发展前景。

《声学技术》编辑部　丁玉薇