Estimation of breakout sound power level due to turbulence caused by an in-duct element

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Abstract: The relationship between the fluctuating drag and lift pressures in the turbulent area within a flow duct is given. Based on the assumption of Nelson and Morfey that there is a constant of proportionality between the fluctuating drag force and the steady state drag force acting on an in-duct element, a pressure-based technique is proposed for estimating the breakout noise due to the vibration of the duct wall caused by the interaction of the turbulent flow and the in-duct elements.

Key words: drag pressure; turbulent flow; breakout noise

管道内部湍流所引起的泄漏噪声之声强级估算

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摘要：在管道通风系统中的湍流区内，震荡波动的阻力气压与推力气压之间的关系已被阐述。根据 Nelson 与 Morfey 的设定，在流体质元所受波动阻力和稳态阻力之比恒定的条件下，文中推广了一种基于气压值的计算方法，用来推算由管道内部因素与湍流作用所引起的管道壁泄漏噪声之声强级。

关键词：阻力压；湍流；泄漏噪声

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1　INTRODUCTION

Nowadays, there are wide ranges of applications and routine engineering methods used to mitigate or predict the industrial, residential and office noises. However, it is still difficult to predict the noise breakout from the air duct of a ventilation system. It is of necessity to develop a method to estimate the sound level and the spectral content of the sound power radiated from the vibrating duct walls at design stage of the ventilation system.

Noise breakout is due to the wall vibration and the production of the wall vibration can be either driven by the internal sound fields or the internal flow turbulence. In this work, attention will be paid to the vibration of duct wall due to the turbulence generated by the interaction of air flow and an in-duct element.

Heller and Widnall[1] conducted experiments to measure the fluctuating drag and lift forces acting on the in-duct element when airflow passes through the air duct. They experimentally demonstrated that there was a constant of proportionality between the

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fluctuating drag force and the steady state drag force acting on an in-duct element. Nelson and Morfey [2] later made this assumption in the derivation of their pressure-based technique for predicting the aerodynamic noise produced by a simple strip spoiler in a low speed flow duct. Oldham and Ukpoho[3] further developed the pressure-based technique of Nelson-Morfey for determining the flow-generated noise caused by a more complex element in an air duct. Mak and Yang[4,5] and Mak[6] later extended the technique to include the acoustic interaction of two in-duct flow elements. This method was further extended by Mak[7] to consider acoustic interactions of multiple in-duct elements. The authors have recently revised the method to include the turbulent interaction of two in-duct elements[8]. In this paper, the relationship of the lift and drag forces and the pressure drop across the in-duct element will be employed to derive a pressure-based method for estimating the breakout noise caused by the wall vibration due to the flow turbulence in the duct. By combining previous works[1-8], a pressure-based technique will be produced to determine both the internally and externally radiated sound powers in duct systems caused by the interaction of the flow and the in-duct element.

2 THEORY

2.1 Work of Heller and Widnall

The fluctuating drag and lift forces acting on the flow elements were measured directly in Heller and Widnall's experiments[1] in order to investigate the influence on the sound radiation within the duct. In their experimental data, it is worthy noticing that the fluctuating lift pressure P_{lift} acting on the ele-ment is approximately equal to the fluctuating drag pressure P_{drag} except at the bending mode frequencies of the strip element (Appendix 1), i.e.

$$P_{lift} \approx P_{drag} \tag{1}$$

In Heller and Widnall's experiments, the element was located at the axis of the duct (that is, the center of the turbulent area). Although the steady state drag pressure is much larger than the steady state lift pressure because of the air flow along the duct acting on the surface of the element, the turbulence generated by the interaction of the flow and the element can be regarded as isotropic when the turbulent area is in a steady state after a short period of time. The fluctuating drag and lift pressures induced by the isotropic turbulence are therefore approximately equal.

2.2 Assumption of Nelson and Morfey

The basis of the pressure-based prediction technique of Nelson and Morfey[2] is that the root mean square fluctuating drag force acting on the element is directly proportional to the steady state drag force $\bar{F}_3$, i.e.

$$(F_{drag})_{rms} = K(St)\bar{F}_3(\text{band } f_c/\alpha \text{ to } f_c\alpha) \tag{2}$$

where St is the Strouhal number, K(St) is the ratio dependent on the choice of α. The equation was experimentally confirmed by Heller and Widnall[1] and then further developed by Oldham and Ukpoho[3]. The steady state drag force acting on the element can be obtained easily by measuring the static pressure difference Δp_s between a location upstream of the element and a location sufficiently far downstream, which allows full static pressure recovery in the fluid after the flow constriction. The mean drag force is determined by $\bar{F}_3 = A\Delta p_s$, where is the area of the duct cross-section.

2.3 Sound radiation by vibrating wall

When pressure p acts on the duct wall, multiple modes of the structural velocity are produced and the velocity spectrum of the mode mn can be calculated by $\bar{v}^2_{mn} = \frac{\pi \Phi_{pmn}(\omega_{mn})}{m_s^2 \eta_s \omega_{mn}}$ [9], where m_s is the mass per unit area, η_s is the loss factor that is the same for all modes, $\Phi_{pmn} = \Phi_{pp}|\Psi_{mn}|^2$ is the auto spectrum of the modal pressure, Φ_{pp} is the pressure spectral density, and the mode shape function $\Psi_{mn}(y)$ satisfies $\iint_{A_p} \Psi_{mn}(y)\Psi_{op}(y)d^2y = \delta_{mnop}A_p$. The total reverberant velocity will be $\bar{v}^2=\int_{\omega_2}^{\omega_1}\bar{V}^2_{mn}(\omega_f)n(\omega_f)d\omega$. For a square air duct, the mode density $n(\omega)=\frac{ab}{4\pi}(\frac{m_s}{D_s})^{1/2}$, where a and b are the dimension of the cross-section and the length of the duct respectively, and D_s is the uniform stiffness. In this way, the power radiated from the duct wall in the radian frequency bandwidth $\Delta\omega$ can be estimated by $W_{rad}(\omega_f,\Delta\omega)=\rho_0c_0A_p\overline{\sigma}(\omega_f)\bar{v}^2$, where $\overline{\sigma}$ is the average radiation efficiency that has been discussed in Ref. [9]. After the some derivations, the equation to calculate the externally radiated sound power from the turbulence-induced vibration is given by

$$W_{rad}\approx\rho_0c_0A_p\overline{\sigma}(\omega_f)\int_{\omega_2}^{\omega_1}\frac{ab\Phi_{pp}(\omega_f)}{4m_s^2\eta_s\omega_f}(\frac{m_s}{D_s})^{1/2}d\omega \tag{3}$$

3 PRESSURED-BASED METHOD FOR SOUND POWER RADIATED EXTERNALLY

When there is an element in a duct, the interaction of the oncoming flow and the obstacle will generate a turbulent area. Assuming the fluctuating lift pressure has no attenuation from the element to the duct wall and the element is at the axis of the duct, the fluctuating lift pressure which vertically acts on the duct wall is regarded as the main drive to induce the wall vibration. Based on equations (1) and (2),

$$(P_{lift})_{rms}\approx K(St)\bar{F}_3/A_S \tag{4}$$

where A_S is the area of the element facing the coming flow, $\bar{F}_3=C_D(\frac{1}{2}\rho_0U_C^2)\sigma^2(1-\sigma)A$, in which C_D is the drag coefficient, σ is the open area ratio and U_C is the constriction velocity[2]. The mean square value of the fluctuating lift pressure in a given bandwidth is

$$(\overline{P^2_{lift}})_{\Delta\omega}=\frac{1}{2}\int_{\omega_2}^{\omega_1}\Phi_{pp}(\omega)d\omega \tag{5}$$

Substituting Eq.(4) into $W_{rad}\approx\rho_0c_0A_p\overline{\sigma}(\omega_f)\frac{ab(P^2_{lift})_{\Delta\omega}}{8m_s^2\eta_s\omega_f}\cdot(\frac{m_s}{D_s})^{1/2}$ which is from Eq.(3), the radiated sound power can be rewritten by

$$W_{rad}\approx\rho_0^3c_0A_p\overline{\sigma}(\omega_f)\frac{ab}{32m_s^2\eta_s\omega_f}\cdot(\frac{m_s}{D_s})^{1/2}K(St)^2C_D^2U_c^4\sigma^4 \tag{6}$$

Furthermore, the externally radiated sound power level SWL_F in 1/3 octaves can be normalized in this way

$$120+20\lg K(St)=SWL_F-10\log_{10}[\frac{\rho_0^3c_0A_p\overline{\sigma}(\omega_f)ab}{32m_s^2\eta_s\omega_f}(\frac{m_s}{D_s})^{1/2}C_D^2U_c^4\sigma^4] \tag{7}$$

It is noted that the area of the vibrating wall induced by the in-duct turbulent area should be $A_p=4aL$ (four plates), where L is the main effective range of the turbulence along the duct axis, and the approximate determination of its L=10a value comes from the experimental data[2]. It is obvious from the equation that the sound power radiated from the vibration of the duct wall is proportional to the three powers of the ambient density ρ_0.

On the other hand, the internal sound pressure causes the wall to vibrate at the same time, and the effect is prominent at low to mid-frequencies[9,10]. The transmission loss is usually defined by $TL=10\log_{10}(\frac{W_i/A}{W_r/A_p})$ [9], and the prediction form for plates is[10].

$$TL\approx10\log_{10}(\frac{m_s^2\omega^2}{12.7\rho_0^2c_0^2}) \tag{8}$$

where W_i and W_r are the sound power within the duct and radiated by the duct wall. The radiated sound power level generated by the sound-induced vibration is obtained by $SWL_F=SWL_D-TL+10\log_{10}\frac{A_p}{A}$, where SWL_D is the internal sound power level which can be obtained from previous works[2,3]. For $f<f_0$

$$120+20\log_{10}K(St)=SWL_D-10\log_{10}(\frac{\rho_0A\sigma^4C_L^2U_c^4}{16c_0}) \tag{9}$$

For $f>f_0$

$$120+20\log_{10}K(St)=SWL_D-10\log_{10}[\rho_0\pi A^2(St)^2\sigma^4C_L^2U_c^6/24c_0^3r^2]-10\log_{10}[1+\frac{860}{f}] \tag{10}$$