A study of 1/f noise

XU Sheng-long

(*Kunming Institute of Physics*, *Kunming* 650223, *China*)

Abstract: On the basis of our previous work it has been found that the 1/f noise is a kind of "phase" with a developing process. And the fact that the 1/f noise can be finally formed or not depends on if the minimum demand for reliability of $M^{\#}>500$ being satisfied. Moreover this is also a criterion for distinguishing 1/f noise. Subsequently it can be learnt that the entropy of 1/f noise is $S^{*}=A\times10^{-20}erg/k$ and S^{*} is a minimum value for all kinds of 1/f noise, which is their general character. But, for different 1/f noise A is a variable value, which represents the individual character of 1/f noise.

Key words: phase; entropy; 1/f noise; self-organization

1 INTRODUCTION

What is the 1/f noise? Searched for 1/f noise on internet no answer could be found. Japanese scientist Gao An Xiu Shu indicated "What is called 1/f noise is the general name of an oscillation whose power spectrum is in inverse proportion to the frequency f",but did not reveal its nature. Our works[1-2] indicated that the origin of 1/f noise is "The surge of back-ground energy" caused by the interaction be-tween system and random action, and solved all the questions about the 1/f noise comple-tely to get the following definition: "1/f noise is a new 'phase', as a highly controlled mov-ement state caused by the surge of background energy under special conditions".

There are really some new ideas in many papers from internet such as "self-organizing", but cannot reach the goal due to something missed in the general train of thought, for example how to decide a successful or a failure "self-organizing"? The other ideas, such as "fluctuation", "trap" and "in-linear", could get some results in describing physical mec-hanisms, but have many limitations.

Therefore it's necessary for 1/f noise to have a rigorous and clear explanation.

The reliability $M^{\#}$ is described in Fig.1. 1/f noise can be formed only when $M^{\#}>H_2$, and the "self-organizing" fails under the four conditions ($\overline{CC}$, $\overline{DD}$, $\overline{EE}$, $\overline{FF}$).

Our works[1-3] explained the origin and condition of 1/f noise. The criterion for generating 1/f noise is

$$M^{\#}=\frac{1}{\sigma_\eta}\left|\log_{10}\frac{f_2}{f_1}\right|>500$$

where $M^{\#}$ is the reliability of experimental data in frequency band $f_1\leqslant f\leqslant f_2$, and

$$\sigma_\eta^2=\frac{1}{N}\left[\eta_1^2+\eta_2^2+\cdots+\eta_N^2\right]$$

where η is the relative error between experimental and theoretical values, and 500 is the reliability of imitating a straight line for experimental data.

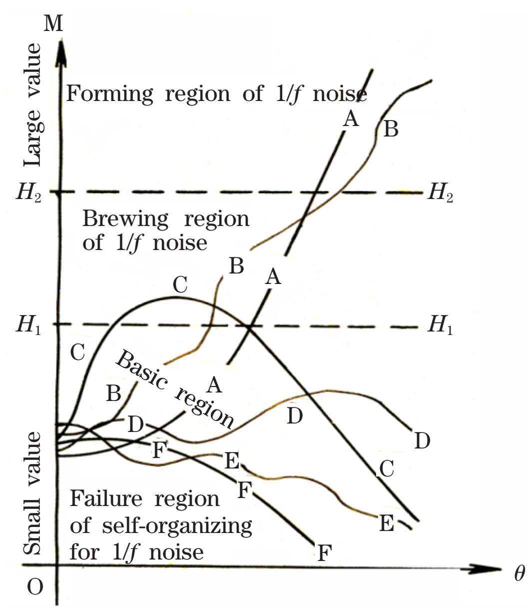

Fig.1 Schematic drawing for possible variation of M#

Received Apr. 20, 2008; Revised Jul. 28, 2008

XU Shenglong, E-mail: xujingquang@126.com

2 FUNDAMENTAL THEORY

The $1/f$ noise was found initially in experiments for a frequency band $f_1 \leqslant f \leqslant f_2$ and the power spectrum follows the relationship

$$S(f) \propto \frac{1}{f^M} \tag{1}$$

The accurate law[1-3] is

$$S(f) \propto \frac{|\lg f|}{f} \tag{2}$$

But for frequency band $f_1 \leqslant f \leqslant f_2$, Equations (1) and (2) is equal almost.

$$\frac{|\lg f|}{f} = \frac{C}{f^M} \tag{3}$$

According to Equation (1) the power spectrum should be a straight line with a slop M in double-logarithmic coordinates shown in Fig.2. The following discussion will start with Equation (1).

Define error

$$E = u - u_t \tag{4}$$

where u is the experimental value (mid-value of measurement), and u_t is the theoretical value. The relative error

$$\eta = \frac{E}{u_t} \tag{5}$$

It is difficult for some experiments to decide whether the experimental data follow Equation (1) Let's discuss from fundamental theory.

The power spectrum of random force $\overrightarrow{\zeta(t)}$ generating $1/f$ noise[1-3] is

$$\vec{B}^*(f)\vec{B}(f) = \frac{\vec{L}^*\vec{L}}{\beta^2 + f^{2\alpha} g(f)} \tag{6}$$

According to Newton second law

$$S(f) = \frac{1}{4\pi f \lambda} \sum_{k=1} \frac{\dfrac{\vec{A}_k^*\vec{L}_k}{m_k}}{\sqrt{\beta^2 + f^{2\alpha} g_k(f)}} \tag{7}$$

If

$$\frac{\vec{A}_k^*\vec{L}_k}{m_k} = C \tag{8}$$

and

$$g_k(f) = g(f) \qquad k = 1, 2, 3, \cdots, N \tag{9}$$

the power spectrum of $1/f$ noise will be

$$S(f) = K \frac{\left| \lg\left[\sqrt{1 + \left(\frac{f}{f_0^*}\right)^{2\alpha} g(f) - 1} \right] \right|}{f} \tag{10}$$

The condition, Equation (9), means that all the character function $g_k(f)$ of random force $\overrightarrow{\zeta(t)}$ in the considering system will be the same. The condition, Equation (8), requires the amplitude of vibration for every point in the system is the same, then the $1/f$ noise must be generated and vice versa, which is like "resonance" in acoustics. Therefore $1/f$ noise is a new special "phase" which hides in physical dynamic states and appears under special conditions.

In order to adapt for all specific cases from macro to micro, from nature to society and from science to art etc., the conditions Equations (8) and (9) are necessary and rigorous to avoid being mislead.

It needs to emphasize that to what extent the general character and their difference reaches as $1/f$ noise performs in different circumstances To take snowflake as an example for comparison, it is no doubt that almost the snowflake looks like having the same size and the same regular hexagon on the whole, but if observe carefully, it can be found that the structure of every piece of snowflake changes in thousands of ways. Similar to this, in the characters of $1/f$ noise there is entirely consistent generality and completely individual difference. In my next paper – 《The entropy of $1/f$ noise》, it can be proved that the value of the entropy of $1/f$ noise is $A \times 10^{-20}$erg/k. Regarding the value of the entropy of $1/f$ noise, the minimal value of 10^{-20}erg/k represents the general character of $1/f$ noise. This is a highly controlled performance of $1/f$ noise. But the different value of A means the individual character of $1/f$ noise. That is, the different $1/f$ noise has its own particular movement state resulting in imperceptible difference of A.

Based on the previous works[1-3] the problem how to soften the conditions Equations (8) and (9) can be discussed, and basically no effect happens on power spectrum of $1/f$ noise. To explain self-organizing process, the power spectrum, Equation (6), of random force $\overrightarrow{\zeta(t)}$ should be expanded as

$$\vec{B}^*(f)\vec{B}(f) = \frac{\vec{L}^*\vec{L}}{\beta^2 + f^{2\alpha} g(f, \theta)} \tag{11}$$

where $g(f)$ in Equation (6) changes into $g(f,\theta)$, which varies with θ, and θ may be a function of temperature, current, press and time etc..

Now, suppose that all $g_k(f,\theta), k=1,2,3,\cdots$ are not completely equal, but the difference of each other is much small.

$$g_1(f,\theta)\cong g_2(f,\theta)\cong\cdots\cong g_N(f,\theta) \tag{12}$$

The average of $g_k(f,\theta)$ is

$$g(f,\theta)=\frac{1}{N}\left[g_1(f,\theta)+g_2(f,\theta)+\cdots+g_N(f,\theta)\right] \tag{13}$$

The error $\varepsilon_k, k=1,2,\cdots N$, are

$$\varepsilon_k(f,\theta)=g_k(f,\theta)-g(f,\theta) \tag{14}$$

$$\frac{\varepsilon_k(f,\theta)}{g(f,\theta)}\approx 0 \tag{15}$$

And the standard error is

$$\sigma^2=\frac{1}{N}\left[\varepsilon_1^2+\varepsilon_2^2+\cdots\varepsilon_N^2\right] \tag{16}$$

Substituting Equation (14) into the sum of Equation (7) gives

$$\frac{1}{\sqrt{\beta_k^2+f^{2\alpha}g_k(f,\theta)}}=\frac{1}{\sqrt{\beta_k^2+f^{2\alpha}(g(f,\theta)+\varepsilon_k(f,\theta))}}$$
$$=\frac{1}{\sqrt{\beta_k^2+f^{2\alpha}g(f)}}+\mu_k(f,\theta) \tag{17}$$

where

$$\mu_k(f,\theta)=\frac{1}{\sqrt{\beta_k^2+f^{2\alpha}g_k(f,\theta)}}-\frac{1}{\sqrt{\beta_k^2+f^{2\alpha}g(f,\theta)}}$$
$$\cong\frac{f^{2\alpha}\varepsilon_k}{2\left[\beta_k^2+f^{2\alpha}g(f,\theta)\right]^{3/2}} \tag{18}$$

With Equations (17), (7) and (15), the power spectrum $S(f)$ of $1/f$ noise has an extra item

$$\sum_{k=1}\mu_k(f,\theta) \tag{19}$$

but it is very small and not to affect $S(f)$. That is to say the Equation (10) is correct while softening the conditions of Equations (8) and (9).

According to classical analysis the fluctuation ratio

$$K(f,\theta)=\frac{\sigma(f,\theta)}{g(f,\theta)} \tag{20}$$

and the reliability

$$M^{\#}(f,\theta)=\frac{g(f,\theta)}{\sigma(f,\theta)} \tag{21}$$

The dynamic statistics theory of $1/f$ noise requires to satisfy the conditions of Equations (8) and (9), in other words requires

$$\sigma(f,\theta)=0 \tag{22}$$

then

$$M^{\#}(f,\theta)\to\infty \tag{23}$$

In fact it is difficult to get Equation (23), therefore the variance of $M^{\#}(f,\theta)$, which may be waved, has to be analyzed. The $1/f$ noise can be formed only when $M^{\#}(f,\theta)$ is great enough, and before that the region of $M^{\#}(f,\theta)$ ranging divides into A: basic, $M^{\#}\leqslant H_1$, B: brewing, $H_1<M^{\#}\leqslant H_2$, and C: forming, $H_2<M^{\#}$. If $M^{\#}$ is not over H_2 the $1/f$ noise cannot be formed, it means that the effect of extra $\sum_{k=1}\mu_k(f,\theta)$ cannot be neglected.

3 EXPERIMENTAL CRITERION

The empirical Equation (1) is shown as a straight line in double logarithmic coordinates. Supposed a set of experimental data (x_k, y_k), $k=1,2,\cdots N$, the least squares fit line can be obtained as in Fig.2 and the calculated relative error isηk and σ_η. The reciprocal of σ_η is the reliability. The criterion displaying $1/f$ noise is supposed as

$$M^{\#}=\frac{1}{\sigma_\eta}\lg\frac{f_2}{f_1}>H_2 \tag{24}$$

For $1/f$ noise in optical spectrum[1,3]

$$S(f)\propto\frac{1}{f^{1.005\pm0.015}}$$

While $2\times10^{-10}\leqslant X\leqslant 5\times10^{-6}$

$$\frac{|\lg X|}{X}=\frac{6.5832}{X^{1.055}}$$

Calculating both sides of the equation the difference is $\sigma_\eta=1.09‰$, and $M^{\#}=4034.8$ is very big.

According to reference [2] there are 5 straight lines (Z_1, Z_2, Z_3, Z_4, Z_5) to imitate $\frac{|\lg X|}{X}$ in the region of $1\times10^{-3}\leqslant X\leqslant 1\times10^{-2}$, the results are

For Z_1 $\sigma_\eta=0.00101$ and $M^{\#}=990$
For Z_2 $\sigma_\eta=0.00109$ and $M^{\#}=915$
For Z_3 $\sigma_\eta=0.00112$ and $M^{\#}=891$
For Z_4 $\sigma_\eta=0.00112$ and $M^{\#}=891$
For Z_5 $\sigma_\eta=0.00116$ and $M^{\#}=856$

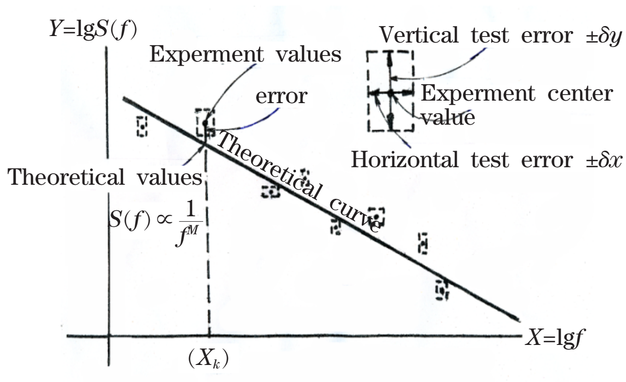

Fig.2 Experiment data and theoretical curve

So, there are little differences among them. Selecting other Z, the results should be the same and can be taken as a criterion.

There is always uncertainty in measurements, so it is necessary to reduce the value H_2, and H_2=500 will be reasonable, of course may be further discussed.

References

[1] XU Shenglong. Statistical dynamic of 1/*f* noise[J]. Infrared Technology, 2007, (9):531-535

[2] 许生龙. 1/*f* 再论噪声的动力学统计理论[J]. 中国测试技术, 2007, (3): 79-83.
XU Shenglong. Re-study of statistical dynamic of 1/*f* noise[J]. China Measurement & Testing Technology, 2007, (3): 79-83.

1/*f* 噪声研究

许生龙

(昆明物理研究所, 昆明 650223)

摘要: 在以往的工作的基础上,发现 1/*f* 噪声是一个新的相。它有一个酝酿、发育的过程,最终能否成形,取决于能否满足可信度 $M^{\#}$ 的最低要求,即 $M^{\#}>500$。这也是判别是否是 1/*f* 噪声的依据。往后还可得知,1/*f* 噪声的熵 S^{*} 值为 $S^{*}=A\times10^{-20}$erg/k(尔格/度)。S^{*} 极小,是所有 1/*f* 噪声的共性。A 不同则是个性。

关键词: 相;熵;1/*f* 噪声;自组织

中图法分类号: TB556 **文献标识码:** A **文章编号:** 1000-3630(2008)-06-0921-04