Research on adaptive equalization algorithm for sparse multipath channel

LI Wen-yan, ZHU Ting-ting, WANG Qi
(*School of Electronic Information Engineering*, *Xi'an Technological University*, *Xi'an* 710021, *Shaanxi*, *China*)

Abstract: In order to solve the problem of poor performance of traditional adaptive equalization algorithm in sparse multipath channels, a new adaptive equalization algorithm based on ℓ_2-norm is proposed. This algorithm takes advantage of the sparsity of equalizer weights in sparse multipath channel, and regards the training process of adaptive equalizer as the weighted sum of sparse signal to dictionary in compressed sensing theory, so as to solve the problems of iterative parameter setting and slow convergence. This new algorithm, which combines ℓ_2-norm with compressed sensing, not only improves the weight accuracy, but also reduces the computational complexity. Simulation results show that the proposed algorithm can achieve better performance with less computation amount and fewer training sequences, and has a reference value for improving the communication performance of the system.

Key words: sparse multipath channel; adaptive equalization; ℓ_2-norm; compressed sensing

0 Introduction

Inter-symbol interference (ISI) caused by multipath propagation is especially common in underwater acoustic communication and broadband mobile communication[1], which has greatly hindered the transmission of reliable information. In order to offset the inter-symbol interference, the filter needs to adaptively track the change in channel conditions to recover the distortion caused by multipath transmission[2] in the moving environment.

Common underwater acoustic communication and broadband mobile communication are generally sparse multipath channels, that is, the energy of channel impulse response mainly focuses on a few taps with a long interval, and the energy of most taps tends to zero. When the source passes through the channel, inter-symbol crosstalk can be up to dozens or even hundreds of symbol intervals.

However, in the sparse multipath channel, the traditional adaptive algorithm needs to send a large number of training sequences periodically, which leads to the slow convergence speed of the algorithm[3]. Therefore, an adaptive algorithm with small computation amount and fast convergence speed is required in the sparse multipath channel. In this paper, a new adaptive equalization algorithm is introduced, which combines the ℓ_2-norm of the equalizer output on the basis of the traditional minimum mean square error algorithm, and then combines compressed sensing due to sparseness of the equalizer weights in the sparse multipath channel. Therefore this algorithm can be called compressed training based adaptive (CoTA) algorithm.

1 Adaptive equalization and compression sensing theory

In digital communication, ISI is caused by the change of channel characteristics, and it can be reduced or eliminated by equalization[4]. Due to the randomness and time variability of mobile fading channels, the equalizer must be able to track the time-varying characteristics of mobile communication channels in real time[5]. Adaptive equalizer generally includes two working modes: training mode and tracking mode[6]. In this paper, the least mean square error criterion is used to realize adaptive equalization. Figure 1 is a simple schematic diagram of an adaptive filter[7], here where total $2N+1$ taps are set up and the corresponding weighting coefficients are $C_{-N}, C_{-N+1}, \cdots, C_N$ respectively.

Assuming that the sampling value sequence of the input waveform is $\{x_k\}$ and the sampling value sequence of the output waveform is $\{y_k\}$, then:

$$y_k = \sum_{i=-N}^{N} C_i x_{k-i}, \qquad k = -2N, \cdots, +2N \tag{1}$$

The sending sequence is defined as $\{a_k\}$ and then the error signal is:

Received: 2019-07-08; **Revised:** 2019-10-10
Fund: Shaanxi province science and technology key research and development program general projects (2019GY-084)
Author: LI Wenyan (1994—), female, was born in Baoji, Shaanxi province, China. She received the master's degree. Her research fields is information transmission and processing.
Corresponding author: LI Wenyan, E-mail: 2963454019@qq.com

$$e_k = y_k - a_k \tag{2}$$

Mean square error is defined as:

$$\overline{e^2} = \mathrm{E}(y_k - a_k)^2 \tag{3}$$

According to (1):

$$\overline{e^2} = \mathrm{E}(\sum_{i=-N}^{N} C_i x_{k-i} - a_k)^2 \tag{4}$$

Thus, the mean square error is a function of the tap gain, and it is expected that the mean square error should be minimized for any k, The partial derivative of the above equation to C_i can be written as:

$$\frac{\partial \overline{e^2}}{\partial C_i} = 2\mathrm{E}[e_k x_{k-i}] \tag{5}$$

where e_k is the error signal the error mainly refers to inter-symbol crosstalk and noise.

To minimize the mean square error, $\mathrm{E}[e_k x_{k-i}]$ must be equal to zero, which requires the error to be independent of the input sample values of the equalizer. In Fig.1, the tap gain can be adjusted by the statistical average of the product of the error and the sample value. If the average is not equal to zero, the gain should be adjusted so that the average value approaches zero until it is equal to zero.

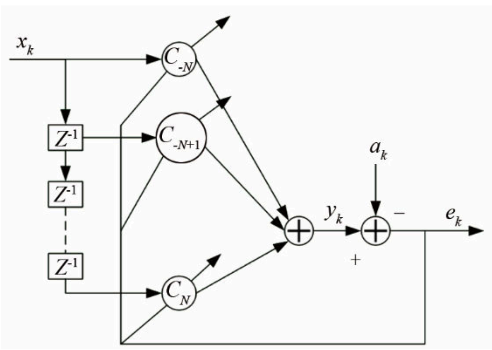


Fig.1　Schematic diagram of adaptive filter

The adaptive convergence of weights in traditional Least Mean Square (LMS) algorithm is slow in sparse multipath channels, where a large number of sequences are required, and the utilization rate of frequency band is not ideal[8]. However, impulse response energy of sparse multipath channel is mainly concentrated on several taps with large spacing between each other, and the most of tap energies are tending to zero, so the weights of the equalizer in sparse multipath channel are sparse, and the training process of adaptive equalizer can be regarded as the weighted sum of sparse signals to the dictionary in Compressed Sensing (CS) theory. Many engineering problems involve the process of solving sparse signals, but the CS theory needs to satisfy two necessary conditions: The original signals are sparse in a transform domain and the observation matrix satisfies finite isometric property[9]. CS can reconstruct potential sparse signals from actual observations by solving optimization problems, and the sparse domain is used to restore the original signals, therefore, it has a wide range of applications in medical, communication, imaging and other scientific fields.

In this paper, the CS method is used to reflect the overall channel g_n which is based on the ℓ_2-norm of the output sequence $\{Z_k\}$ of the equalizer. That is, the output of the peak equalizer is a measure of the sparseness of g_n: the lower the output of the peak equalizer, the sparser the equalization channel.

The relationship between the output $\{Z_k\}$ of equalizer and the emission symbol $\{S_n\}$ and the impulse response $\{g_n\}$ of equalized channel is shown as follows:

$$z_n = \sum_{k=0}^{L_G-1} g_k s_{n-k} \tag{6}$$

Thus, it can be concluded that:

$$|z_n| = \left|\sum_{k=0}^{L_G-1} g_k s_{n-k}\right| \le \sum_{k=0}^{L_G-1} |g_k s_{n-k}| \le \|g\|_2 \|s_n\|_\infty \tag{7}$$

The above inequality is caused by triangle inequality and Holder inequality[10] respectively, and the inequality $\|z_p\|_\infty \leqslant \|g\|_2 \|s_n\|_\infty$ can be directly obtained from Holder inequality. If the M-PAM constellation is assumed to be used for transmitting symbols, namely: $s_n \in \{-M+1, -M+3, \cdots, M-3, M-1\}$, then $\|s_n\|_\infty \leqslant (M-1)$. Therefore, the following bound can be obtained:

$$\|z_P\|_\infty \leqslant (M-1)\|g\|_2 \tag{8}$$

$$z_l = \sum_{k=0}^{L_G-1} g_k s_{l-k} = \sum_{k=0}^{L_G-1} g_k sign(g_k)(M-1) = (M-1)\sum_{k=0}^{L_G-1} |g_k| = (M-1)\|g\|_2 \tag{9}$$

The above equation indicates that under a suitable assumption, the inequality (8) can be written as an equality, given that the transmitting sequence is sufficiently rich in the sense.

If there is any l to make: $s_{l-k} = \mathrm{sign}(g_k)(M-1)$ $\forall k \in \{0, \cdots, L_G-1\}$ or $s_{l-k} = -\mathrm{sign}(g_k)(M-1)$ $\forall k \in \{0, \cdots, L_G-1\}$ true, then $z_l = -(M-1)\|g\|_2$ is also true. According to the expression (8), it can be concluded that:

$$\|z_P\|_\infty = |z_l| = (M-1)\|g\|_2 \tag{10}$$

The ℓ_∞-norm of the equalizer output vector reflects the ℓ_2-norm of the equalized channel impulse response. As a result, the minimization of the peak absolute value of the equalizer outputs amounts to the sparsification of the combined channel equalizer impulse response or equivalently to the reduction of the ISI.

2 Compression sensing adaptive equalization settings

In the complex square Multiple-Quadrature Amplitude Modulation (M-QAM) constellation case, the transmitting symbols take their values from the set $Q=\{a+jb:a,b\in\{-\sqrt{M}+1,-\sqrt{M}+3,\cdots,\sqrt{M}-3,\sqrt{M}-1\}\}$. The optimization settings for Pulse Amplitude Modulation (PAM) can be adapted to the complex QAM scenario by replacing z_P with the vector $\tilde{z}_P \triangleq [\mathrm{Re}\{z_P^{\mathrm{T}}\}\ \mathrm{Im}\{z_P^{\mathrm{T}}\}]^{\mathrm{T}}$. For example, the optimization in Noiseless-Setting for QAM can be converted simply as Noiseless-Setting-QAM

$$\underset{W}{\text{minimize}}\ \|\tilde{z}_P\|_\infty \quad \text{subject } to\ Y_T w=s_T \tag{11}$$

and similarly *Noisy-Setting-III* can be adapted to the QAM case as Noiseless-Setting-III-QAM:

$$\underset{W}{\text{minimize}}\ \|Y_T w-s_T\|_2+\lambda\|\tilde{z}_P\|_\infty \tag{12}$$

The reason behind this slight modification of the QAM case can be explained by the following equation:

$$\begin{bmatrix}\mathrm{Re}\{z_n\}\\ \mathrm{Im}\{z_n\}\end{bmatrix}=\begin{bmatrix}\mathrm{Re}\{g\}^{\mathrm{T}} & -\mathrm{Im}\{g\}^{\mathrm{T}}\\ \mathrm{Im}\{g\}^{\mathrm{T}} & \mathrm{Re}\{g\}^{\mathrm{T}}\end{bmatrix}\begin{bmatrix}\mathrm{Re}\{s_n\}\\ \mathrm{Im}\{s_n\}\end{bmatrix} \tag{13}$$

Consequently,

$$\begin{aligned}\|\tilde{z}_P\|_\infty &= \|[\|\mathrm{Re}\{z_P\}\|_\infty\ \ \|\mathrm{Im}\{z_P\}\|_\infty]\|_\infty\\ &(\sqrt{M}-1)\|\tilde{g}\|_2\end{aligned} \tag{14}$$

As a result, the minimization of $\|\tilde{z}_P\|_\infty$ amounts to the sparsification of $\tilde{g}$ and, therefore, the corresponding g, as in real PAM constellations. Furthermore, the equivalent optimization settings for QAM can be written in terms of $\tilde{g}$. As an example, the equivalent of Noiseless-Setting-QAM in this case can be written as Noiseless-Equivalent-Setting-QAM:

$$\underset{\tilde{g}}{\text{minimize}}\ \|\tilde{g}\|_1 \text{ subject to} \begin{bmatrix}\mathrm{Re}\{S\} & -\mathrm{Im}\{S\}\\ \mathrm{Im}\{S\} & \mathrm{Re}\{S\}\end{bmatrix}\tilde{g}=\tilde{s}_T \tag{15}$$

In this paper, the adaptive equalization algorithm combines the ℓ_2-norm output of the equalizer on the basis of the traditional minimum mean square error algorithm, and then combines compressed sensing to form the compressed training based adaptive (CoTA) algorithm. The steps are as follows:

(1) Equalizer outputs: $z[i]\leftarrow Y_P w[i]$;

(2) Real equalizer output peaks:

$P_{\mathrm{re}}[i]\leftarrow\sum_{n=1}^{L_P} 1_{|(\tilde{z}_P[i])_n|>\alpha\|\tilde{z}_P[i]\|_\infty}\mathrm{sign}(\tilde{z}_P[i])_n)e_n$;

(3) Imag equalizer output peaks:

$P_{\mathrm{im}}[i]\leftarrow\sum_{n=L_p+1}^{2L_P} 1_{|(\tilde{z}_P[i])_n|>\alpha\|\tilde{z}_P[i]\|_\infty}\mathrm{sign}(\tilde{z}_P[i])_n)e_{n-L_p}$;

(4) 2- norm gradient: III

$u_2[i]\leftarrow\dfrac{Y_T^{\mathrm{H}}(Y_{TW[i]-S_T})}{\|Y_T^{\mathrm{H}}(Y_{TW[i]-S_T})\|_2}$;

(5) ∞-norm weighted subgradient:

$u_\infty[i]\leftarrow\dfrac{\Pi^{-1}Y_P^H(P_{\mathrm{re}}[i]+jp_{\mathrm{im}}[i])}{\|\Pi^{-1}Y_P^H(P_{\mathrm{re}}[i]+jp_{\mathrm{im}}[i])\|_2}$;

(6) Update vector:

$u[i]\leftarrow w[i]-\mu[i](u_2[i]+\lambda u_\infty[i])$;

(7) Nesterov step:

$w[i+1]=u[i]+\dfrac{i-1}{i+2}(u[i]-u[i-1])$;

(8) Normalization:

$w[i+1]\leftarrow\dfrac{w[i+1]Y_T^{\mathrm{H}}{}_{S_T}}{w[i+1]Y_T^{\mathrm{H}}Y_{TW[i+1]}}w[i+1]$.

In this algorithm, Step 1 is to calculate the output of the equalizer. Step 2～3 are used to determine the potential real and imaginary peaks of the equalizer outputs. Here $\|\tilde{z}_P[i]\|_\infty$ represents the absolute sample peak of the cascaded real and imaginary components of the equalizer outputs[11], Due to the presence of noise, the true peak may do not overlap with the sample peak. As a result, the operation performed at these steps marks all indices of $\tilde{z}_P$ for which the absolute values are within the α (an empirical algorithm parameter) factor of the sample peak as potential peak locations[12]. Therefore, the n^{th} component of $P_{\mathrm{re}}[i](p_{\mathrm{im}}[i])$ is set to the sign of the real (imaginary) part of the n^{th} component of $z_P[i]$ if its absolute value is greater than $\alpha\|\tilde{z}_P[i]\|_\infty$; otherwise, it is set to zero. Step 4 is the gradient for the ℓ_2-norm component, and Step 5 is the weighted subgradient for the ℓ_∞-norm component of Noisy-Setting III-QAM. Steps 6-7 correspond to an accelerated equalizer vector update based on the “Nesterov Method”, where u is an intermediate algorithm variable with $u[0]=0$ initialization. Step 8 is the normalization of the equalizer vector to reduce the bias.

The CoTA algorithm can be used to realize the flexible structure of decision guidance pattern. After the initial iteration, the algorithm can continuously extend the training region by appending the reliable decisions. In the following part, the compressed sensing adaptive equalization without noise is analyzed to judge the impact of training sequence on the results.

2.1 Noiseless case

General communication groups contain evenly distributed M-QAM (or M-PAM) information symbols and the training symbols constructed from the corners of the same constellation. Then, the corresponding noiseless equivalent setting has e_{d+1} as the unique solution, and its probability of P has to be satisfied:

$$P>1-(L_G-1)C^{-L_T+1} \tag{16}$$

Where C is 4 for QAM and 2 for PAM constellations. Equation (16) is also true for general PAM and QAM constellations. Equation (16) illustrates the phase transition result for the sparse reconstruction problem in compressed sensing as it arises as the equivalent problem to original adaptive equalization problem in noiseless setting.

Therefore, the amount of training required in the noiseless (or high-SNR) case is approximately equal to $\log_C(L_G)+\rho$, where ρ represents a safety margin for the completion of phase transition. Assuming that the length of equalizer L_E is approximately equal to the channel spread L_C, then:

$$L_T \approx \log_C(2L_E)+\rho \tag{17}$$

According to the number of equalizer coefficients, for channel propagation:

$$L_T \approx \log_C(2L_C)+\rho \tag{18}$$

This corresponds to a low training size relative to the number of equalizer coefficients, which is equal to rL_E, especially if the channel and equalizer lengths are relatively long.

Therefore, in the case of noiseless, the proposed approach reduces the required training length to less than the number of unknown parameters rL_E, while maintaining the perfect equalization solution $g=e_{d+1}$.

2.2 Noisy case

When SNR is relatively low, the following two performance indicators need to be considered.

ISI level:

$$L_{\mathrm{ISI}}=\mathrm{E}(\|g-e_{d+1}\|_2^2) \tag{19}$$

Mean square error:

$$\begin{aligned} e_{\mathrm{MS}} &= \mathrm{E}(|z_n-s_{n-d}|^2) \\ &= \mathrm{E}(\|g-e_{d+1}\|_2^2)+\mathrm{E}(\|V_T w\|_2^2)L_T^{-1} \end{aligned} \tag{20}$$

Where, the second term on the right hand side of (15) represents the filtered noise power at the output of the equalizer.

It can be seen that L_{ISI} and e_{MS} expressions in the noisy case also confirm that the training length should be greater than $\log(L_G)$.

3 Adaptive single-carrier frequency-domain equalization

The following compressed training approach to Single-Carrier Frequency-Domain Equalization (SC-FDE), takes advantage of the special convolution structure of the frequency-selective channels. In this scheme, the linear convolution channel is, in effect, converted to a circulant convolution channel through the inclusion of a cyclic-prefix symbol. A predetermined Unique-Word (UW) sequence $\{u_n, n\in\{1,\cdots,L_T\}\}$, where L_T is the UW length, is appended to the beginning and the end of the data sequence $\{d_n, n\in\{1,\cdots,L_D\}\}$, where L_D is the length of the data symbols, to form the transmit block of the SC-FDE system. The unique word serves both as the training sequence and as the cyclic-prefix block, and its size is selected to be greater than the presumed channel spread. According to this figure, at each receiver branch, after prefix removal, $L_P=L_T+L_D$ consecutive time-domain receiver samples are vectorized $y^{(k)}\in C^{LP}, k=1,\cdots,r$,and converted to frequency-domain vector $y^{(k)}$ through L_P-point FFT operation, represented as $\boldsymbol{Y}^{(k)}=\boldsymbol{F}^{\mathrm{H}}y^{(k)}$. The equalizer operation corresponds to the combination of elementwise-multiplied frequency-domain vectors

$$\boldsymbol{Z}=\sum_{k=1}^{r}\boldsymbol{W}^{(k)}\otimes\boldsymbol{Y}(k) \tag{21}$$

Where $\otimes$ is the elementwise multiplication operator and $\boldsymbol{W}^{(k)}, k=1,\cdots,r$ are equalizer coefficient vectors. The equalizer output converted to the time domain is given by $\boldsymbol{z}=\boldsymbol{F}\boldsymbol{Z}$.

In the adaptive setting, the scenario where the equalizer is trained by using a single block is considered. This is a desired performance for the adaptive algorithm which gives the channel coherence time constraints mandated by wireless mobile environments. The compressed training approach is a good fit for this task, where the goal is to increase the room for the data symbols by restricting the amount of training symbols in the same block (the presented approach can be easily extended to multi-block-based training). For the adaptive compressed training-based SC-FDE, the following optimization setting is designed:

$$\begin{aligned} &\text{SC-FDE Setting: } \min_{W} \ \|\boldsymbol{E}_d\boldsymbol{z}-\boldsymbol{u}\|_2+\lambda\|\tilde{z}\|_\infty \\ &\qquad\qquad \text{subject to } \ \boldsymbol{W}\in\boldsymbol{F}_{L_E} \end{aligned} \tag{22}$$

Where d is the target equalization delay and $\boldsymbol{E}_d$ is the matrix that extracts the part of the $\boldsymbol{z}$ matrix corresponding the UW symbols based on the selected choice of delay. $\boldsymbol{E}_d$ is a submatrix obtained by deleting L_D rows of the $L_P \times L_P$ identity matrix. It can be written as $\boldsymbol{E}_d = [0 \quad \boldsymbol{I}_{L_T}]\boldsymbol{A}_d$, where multiplication of $\boldsymbol{z}$ with $\boldsymbol{A}_d$ is equivalent to applying the circular d -advance operation on z to position the elements of $\boldsymbol{z}$ corresponding to UW to the last L_T rows, i.e., the compensation of the equalization delay. Multiplication by $[0\ \boldsymbol{I}_{L_T}]$ extracts the UW region from the circularly shifted $\boldsymbol{z}$.

$\mathcal{F}_{L_E}$ is the set of frequency domain equalizers with impulse response length less than or equal to L_E, which can be written as

$$\mathcal{F}_{L_E} = \{W \in C^{L_P} : [0_{L_P - L_E \times L_E} \quad I_{L_P - L_E}]FW = 0\} \tag{23}$$

First its argument is transformed to the time domain, all components are become zeros except the first L_E and then the frequency domain is transformed back to.

4 Simulation experiment and analysis

In order to verify the performance of the proposed compressed training based adaptive algorithm, the traditional LMS adaptive equalization algorithm is compared. The contrast experiments are conducted in the cases with noise and without noise to show the superiority of compressed training based adaptive algorithm.

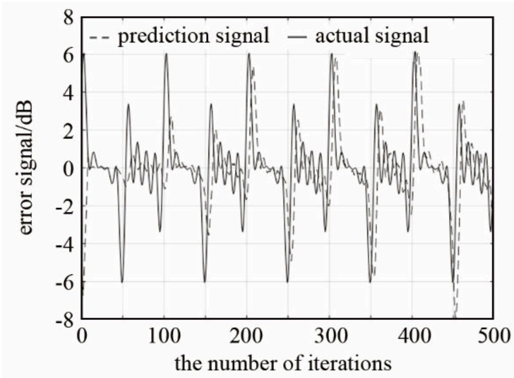
4.1 Noisy case

In this case, the input signal is set as

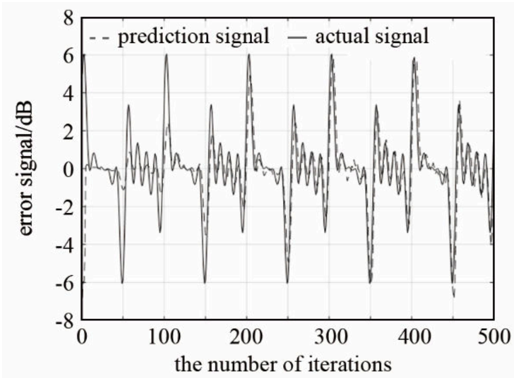
$$\begin{aligned} x(n) = {} & \cos(\pi\times0.02n)+\sin(2\pi\times0.02n)+ \\ & \cos(3\pi\times0.02n)+\sin(4\pi\times0.02n)+ \\ & \cos(5\pi\times0.02n)+\sin(6\pi\times0.02n)+ \\ & \cos(7\pi\times0.02n)+\sin(8\pi\times0.02n)+ \\ & \cos(9\pi\times0.02n)+\sin(10\pi\times0.02n), \quad n=0,\cdots,N-1 \end{aligned}$$

and Gaussian noise with a variance of 0.5 is added to conduct experiments of two different algorithms respectively, in which the number of fixed taps of the compressed training based adaptive algorithm is 5. The experimental results are shown in Fig.2.

From Fig.2(a), it can be seen that the fitting of the actual signal and prediction signal output by the traditional LMS adaptive equalization algorithm is not very ideal and the rate of convergence is slow. The fitting has not converged at the 500$^{\text{th}}$ step of iteration, and even the fluctuation is relatively large, which is mainly caused by the insufficient training sequence. Compared with the traditional LMS adaptive equalization algorithm, the compressed training based adaptive algorithm in Fig.2 (b) has an ideal fitting with the prediction signal and basically converges in about 150 steps of iteration. Its convergence speed is obviously much faster than that of the traditional LMS algorithm.



(a) Traditional LMS adaptive equalization algorithm



(b) Compressed training based adaptive algorithm

Fig.2 Experimental results of adaptive equalization with noise

4.2 Noiseless case

The above signals are still used in this experiment, except that there is no noise added in this case. The experiment results of two different algorithms are shown in Fig.3.

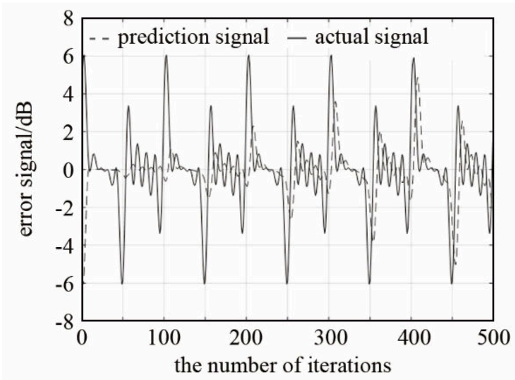
Similar to the results of the experiment 4.1, the convergence speed of the compressed training based adaptive algorithm is much faster than that of the traditional LMS algorithm. The fitting shown in Fig.3(a) does not converge well even at the 500$^{\text{th}}$ step of iteration, while the fitting shown in Fig.3(b) is basically stable at the 150$^{\text{th}}$ step of iteration.

Therefore, whether there is noise or not, the performance of the CoTA algorithm is better than that of traditional LMS algorithms.

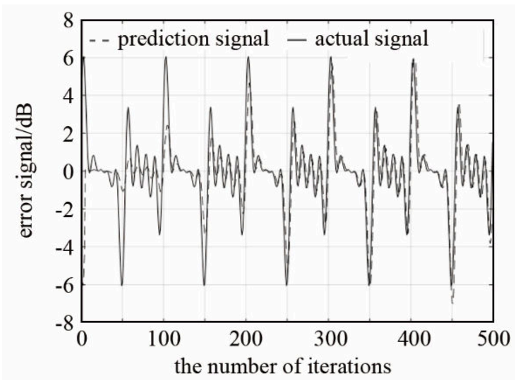
4.3 Comparison with BP algorithm

In order to better illustrate the advantages of the compression equalization adaptive algorithm, the algorithm complexity and precision are compared.

In Table.1 the numbers of iterations required by the CoTA algorithm and the Basis Pursuit(BP) algorithm to achieve optimal performance are listed together with the time complexity of each iteration and the time required to complete the iteration. As shown in Table.1, the time complexity of each iteration of BP algorithm and CoTA algorithm is $O(N)$. However, in order to achieve steady state, the number of iterations of BP algorithm increases by about 300 compared with that of CoTA algorithm. Therefore, CoTA algorithm has a lower number of iterations and a shorter running time.



(a) Traditional LMS adaptive equalization algorithm



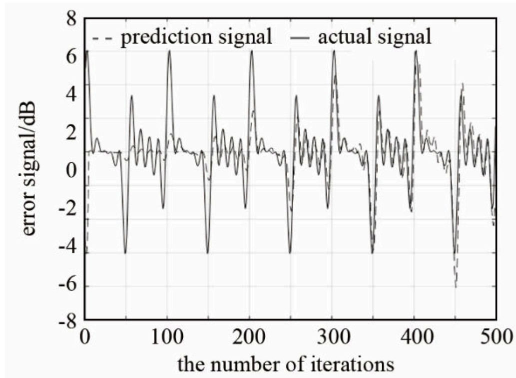
(b) Compressed training based adaptive algorithm

Fig.3 Experimental results of adaptive equalization without noise

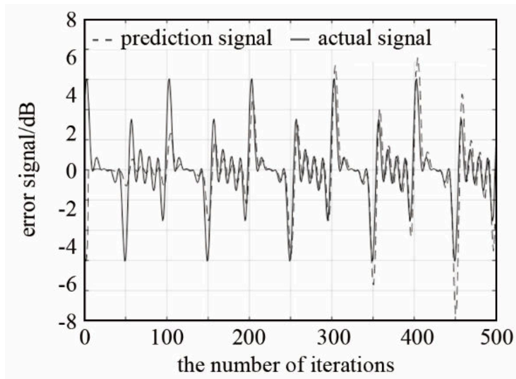
Table 1 Complexity comparison

Algorithm	Time complexity per iteration	The number of iterations	Running time/s
The BP	$O(N)$	426	0.011 8
CoTA	$O(N)$	108	0.009 6

The simulation experiments of the BP algorithm in the cases of the first 500 sample points with and without noise are conducted, and the results are shown in Fig.4, from which it can be seen that no matter whether there is noise or no noise, a large error exists between the actual output signal and the ideal output signal after the 500th step of iteration. Especially, when the signal changes rapidly, the error is more serious. which indicates that the convergence accuracy of the BP algorithm is not high and the convergence effect is not very good even at the 500th step of iteration due to the existance of relatively large fluctuation, which is mainly caused by insufficient training sequences. For the BP algorithm, at least 400 iterations are needed to gradually reach a convergence state, where the actual output signal and the ideal output signal are basically fitted, and the error vector amplitude EVM of the error signal obtained by the simulation is 0.113 2. Such a fitting result is not ideal for modeling in practical application.



(a) noise



(b) No Noise

Fig.4 Experimental results of adaptive equalization with/without noise by BP algorithm

Compared with the experimental results for CoTA algorithm in Fig.2, it can obviously be found that the convergence speed of CoTA algorithm is faster than that of BP algorithm. By the CoTA algorithm, only 90 iterations are required to achieve a better fitting and reasonable output signal. The error vector amplitude EVM of the output signal obtained by the simulation is 0.006 8, which is two orders of magnitude smaller than the EVM obtained by BP algorithm.

The comparison between CoTA algorithm and Basis Pursuit (BP) algorithm can explain why this paper uses ℓ_2 -norm instead of ℓ_1 -norm. BP is a ℓ_1 -norm based algorithm which is much more com-plex than the ℓ_2 -norm based CoTA algorithm pro-posed in this paper. Therefore, the (CoTA) algorithm can not only reduce the computational complexity, but also improve the weight accuracy.

5 Conclusion

In this article, aiming at the shortcoming of slow convergence speed of traditional adaptive equalization algorithm, the compressed training based adaptive algorithm is introduced as a novel approach utilizing the magnitude boundedness of digital communication sources. The algorithm is based on the minimization of the“ ℓ_2-norm” of the equalizer output and a fixed tap constraint is added to the equalizer coefficients. In addition, compressed sensing and adaptive equalization problems are related. It can be seen from the simulation results that the algorithm proposed in this paper improves the training speed and achieves better equalization performance. Therefore, this algorithm has obvious advantages in improving the communication performance of the system under sparse multipath channel.

References

[1] VLACHOS E, LALOS A S, BERBERIDIS K. Stochastic gradient pursuit for adaptive equalization of sparse multipath channels[J]. IEEE Journal on Emerging & Selected Topics in Circuits & Systems, 2012, **2**(3): 413-423.

[2] ZHANG K, HONGYI Y U, YUNPENG H U, et al. Reduced constellation equalization algorithm for sparse multipath channels based on sparse bayesian learning[J]. Journal of Electronics & Information Technology, 2016.

[3] 周孟琳，陈阳，马正华. 一种适用于稀疏多径信道的自适应均衡算法[J]. 电讯技术, 2019, **59**(3): 266-270.
ZHOU Menglin, CHEN Yang, Ma Zhenghua. An adaptive equalization algorithm for sparse multipath channels[J]. Telecommunications technology, 2019, **59**(3): 266-270.

[4] YILMAZ B B, ERDOGAN A T. Compressed training adaptive equalization[C]//In Proc. IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP' 16). IEEE, Mar. 2016: 4920-4924.

[5] OYMAK S. “Convex relaxation for low-dimensional representation: Phase transitions and limitations,” Ph.D. dissertation, California Institute of Technology, Jun. 2015.

[6] 宫改云，姚文斌，潘翔. 被动时反与自适应均衡相联合的水声通信研究[J]. 声学技术, 2010, **29**(2): 129-134.
GONG Gaiyun, YAO Wenbin, PAN Xiang. Research on underwater acoustic communication combined with passive time-inverse and adaptive equalization[J]. Technical Acoustic, 2010, **29**(2): 129-134.

[7] JIAO J, ZHENG X J . Extended sparse multipath channel capacity estimation based on adaptive array configuration[J]. Advanced Materials Research, 2013(765-767): 2728-2731

[8] Al-Awami A T, Azzedine Zerguine, Lahouari Cheded, et al. A new modified particle swarm optimization algorithm for adaptive equalization[J]. Digital Signal Processing, 2011, **21**(2): 195-207.

[9] DONOHO D L. Compressed sensing, IEEE Transactions on Information Theory, 2006, **52**(4), 1289-1306.

[10] 马思扬，王彬，彭华. l_0-范数约束的稀疏多径信道分数间隔双模式盲均衡算法[J]. 电子学报, 2017, **45**(9): 2302-2307.
MA Siyang, WANG Bin, PENG Hua. Sparse multipath channel fractional interval blind equalization algorithm with l_0-norm constraint [J]. Acta electronica sinica, 2017, **45**(9): 2302-2307.

[11] CEVHER V, BECKER S, SCHMIDT M. Convex optimization for big data: Scalable, randomized, and parallel algorithms for big data analytics[J]. IEEE Signal Processing Magazine, 2014, **31**(5): 32-43.

[12] 马丽萍. 多模盲均衡算法的稳态性能研究[D]. 大连: 大连海事大学, 2018.
MA Liping. Research on steady-state performance of multi-mode blind equalization algorithm[D]. Dalian: Dalian maritime university, 2018.

稀疏多径信道自适应均衡算法研究

李文艳，朱婷婷，王 琪
(西安工业大学电子信息工程学院，陕西西安 710021)

摘要：针对传统自适应均衡算法在稀疏多径信道中性能较差的问题，提出了一种基于 ℓ_2-范数的自适应均衡算法。该算法利用稀疏多径信道下均衡器权值的稀疏性，将自适应均衡器的训练过程看作压缩感知理论中稀疏信号对字典的加权求和，以解决迭代参数的设置及收敛速度慢的问题。该算法将 ℓ_2-范数和压缩感知相结合，不仅提高了权值的精度，而且降低了计算复杂度。仿真结果表明，该算法计算量小，训练序列少，具有较好的性能，对提高系统的通信性能具有参考价值。

关键词: 稀疏多径信道；自适应均衡；ℓ_2-范数；压缩感知
中图分类号：TN911 **文献标识码**：A **文章编号**：1000-3630(2019)-06-0698-07
DOI 编码：10.16300/j.cnki.1000-3630.2019.06.017